MULTIDIMENSIONAL TOPOLOGY TRANSFORMS

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SUMMARY

Most currently constructed 3D geological models are to a first order the result of transformations of data with different dimensionality into 3D:

- 0D (e.g. outcrop data, at the regional scale),
- 1D (e.g. drill hole data, at the mine scale),
- 2D data (e.g. satellite data, at the regional scale) or
- 3D data (e.g. seismic data, when high resolution 3D geophysical data are available, such as in basins),
- 4D models (3D evolutions with time).

The datasets used to project between dimensions vary according to the scenario, however they generally consist of a mixture of 0D observations and local temporal or spatial relationships (their topology). Modern software systems are able to use a sub-set of these relationships (fault-stratigraphy relationships for example) to build 3D geological models, however the relationships are not typically used as an independent constraint on how much of the 3D model is constrained by observations, and how much is generated by the end user (or the algorithms they use).

This study explores the relationships between topological observations in 1, 2 and 3D in order to better understand how these may be used in the future as inputs to a revised 3D modelling workflow. We have investigated both synthetic cases, where we have full control, and natural examples, which permit alternate hypotheses. This approach has potential relevance to mine-scale and regional 3D models where the 3D topologies are poorly defined by the existing data, but 1D and 2D constraints are available.

Key words: 3D geological modelling, Topology, Value of Information

INTRODUCTION

What do we mean by topology, VOI, uncertainty, topological uncertainty?

Simple examples and applications of topology in 0D, 1D, 2D, 3D.

The use of spatial and temporal relationships between geological features goes to the heart of ability to reconstruct geological histories. The concepts of stratigraphic relationships and superposition of structures allow us to translate outcrop-scale observations into regional interpretations. The first quantitative analysis of geological maps to extract temporal and spatial relationships was performed manually, which reflects the lack of fully developed methods for describing maps digitally at the time (Burns 1975; Burns et al., 1978; Burns, 1981). A similar, but still manual, approach was taken by Potts & Reddy (1999). With the advent of digital storage of geological data, together with the production of geological maps and 3D models in digital formats, we now have the ability to analyse these data and models for a wide variety of purposes (tectonic, resource analysis, seismic risk etc.).

One of the challenges we face when analysing geological data and models is that it can be collected and presented in a variety of different dimensionalities:

0D data (e.g. outcrop data, at the regional scale),
1D data (e.g. drill hole data, at the mine scale),
2D data (e.g. satellite data, at the regional scale) or
3D data (e.g. seismic data, when high resolution 3D geophysical data are available, such as in basins).
4D models (3D evolutions with time).

This results in challenges in integrating different forms of data, for example integrating a single seismic section with regional potential field geophysical data. It also raises problems when considering the Value of Information associated with different types of models (Eidsvik et al., 2015).

We present a new approach to comparing variable dimensional data and models based on their spatial topology, i.e. the spatial relationships between adjacent lithological units (Thiele et al., 2016a & b).

METHOD AND RESULTS

In this study we define a reduced spatial topology compared to the work of Thiele et al. (2016a & b) as we only consider the adjacency relationships between lithological units, and not the nature of the interface (fault, stratigraphic or intrusive) that separates them (Fig. 1). For 1D data such as drill hole logs, only 2 along-core neighbourhood relationships are be considered: above and below the point of interest. For 2D maps 4 neighbouring pixels are considered for rasterised maps, although higher order neighbourhood relationships that take into account diagonal relationships could be considered. For 3D models the 6 neighbouring voxels are considered, although again 26 relationships could have been considered.

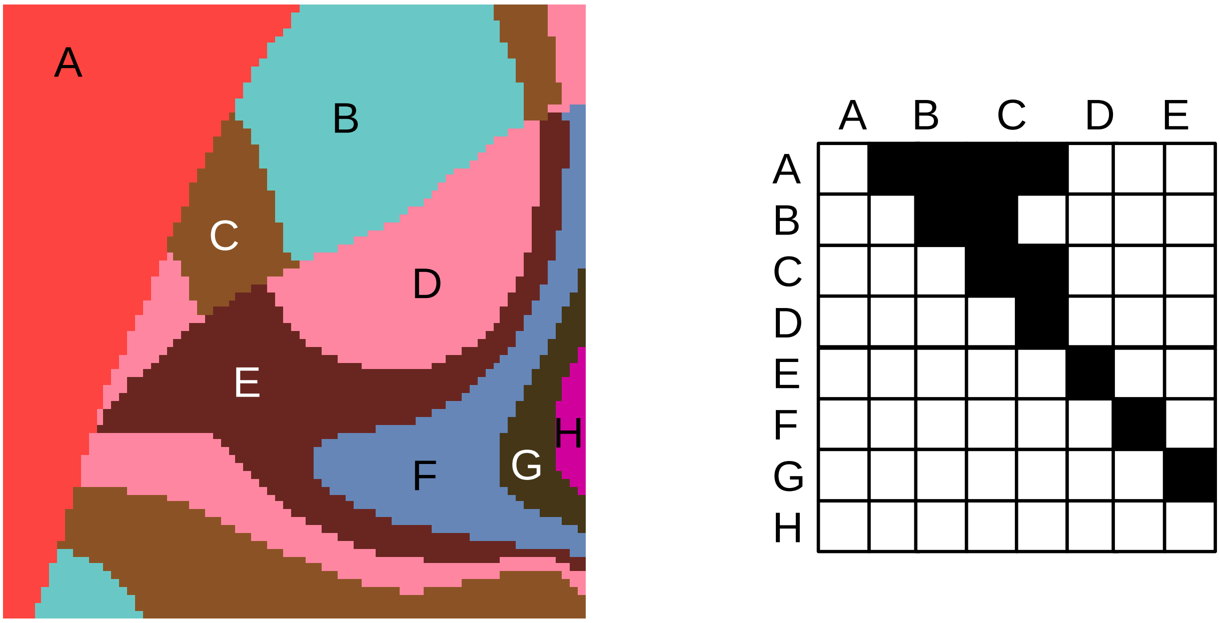

Figure 1: Raster geological map (left) and equivalent Adjacency Matrix (right) showing neighbourhood relationships for each lithological unit. In this case the calculation simply returns a Boolean "are neighbours" (black) or "are not neighbours" (white). The calculation can be performed on raster maps, as shown here) or polygonal GIS files, the only difference being the limits to resolution of individual pixels and the spacing between nodes in the two cases.

As a test case we have used the 3D geological model of South-West Ghana developed by Perrouty et al., (2014). This model was developed as a result a series of implicit and explicit forward and inverse modelling approaches, but for our purposes only the final voxel mode is considered. This model defined 23 distinct lithological units and had dimensions of 160 x 160 x 15 km with a voxel size of 200 m resulting in a model with over 40 million voxels (Fig. 2).

Adjacency matrices were calculated for the complete 3D model with 6 neighbours per voxel (Fig 3 left), the top layer of the model using 4 neighbours per voxel (Fig. 3 middle) and a series of vertical columns of voxels with 2 neighbours per voxel, which we use to simulate vertical drill holes with on a grid pattern with increasing spacing between grid points (Fig. 3 right). We can see that the top layer and the 200 m spaced drill holes (i.e. one drill hole drilled vertically down through the centre of every voxel on the top layer) retain much of the topological information of the full 3D model. As drill-hole spacing increases the Adjacency Matrices become increasingly sparse.

A further comparison can be made between the top layer of the model (Fig. 4 left) and the previously compiled geological map (Fig. 4 right). As one would expect there is much more detail evident in the map than we are able to retain in the 3D model, given the quantity of surface information available relative to sub-surface information in this region.

We can compare the topological information content of the 3D, 2D and 1D models as a function of the number of elements (voxels or pixels) that are used for comparison (so the number of voxels for the full 3D mode, the number of voxels/pixels of the two 2D maps and the summed number of voxels for each grid of simulated drill-holes. In this comparison we simply sum the number of "are neighbour" elements in each Adjacency Matrix (Fig. 5). For the 1D case the number of distinct topologies drops of linearly (in linear-log space) up to 2000 m spacing, and then drops off at a steeper, but still linear, angle beyond that spacing. We are also able to compare the 2D models (top layer and full map) and 3D model. The model with the highest number of distinct topologies is actually the 2D map, followed by the 3D model and finally the top layer of this model.

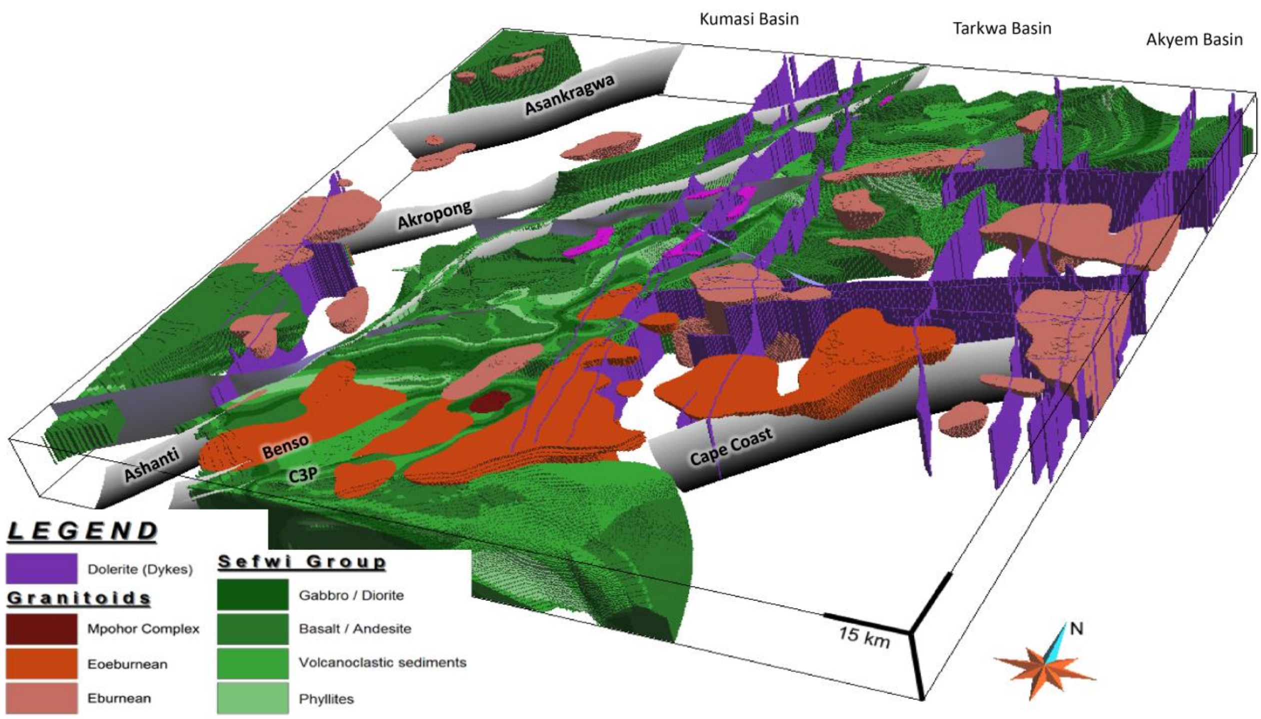

Figure 2: 3D geological model of South West Ghana based on field observations, prior mapping and gravity inversions (after Perrouty et al, 2014). The full 23 lithological units are here simplified to 8 visible units with the others made transparent to aid visualisation. The faults were used to build the model but are not considered in this study.

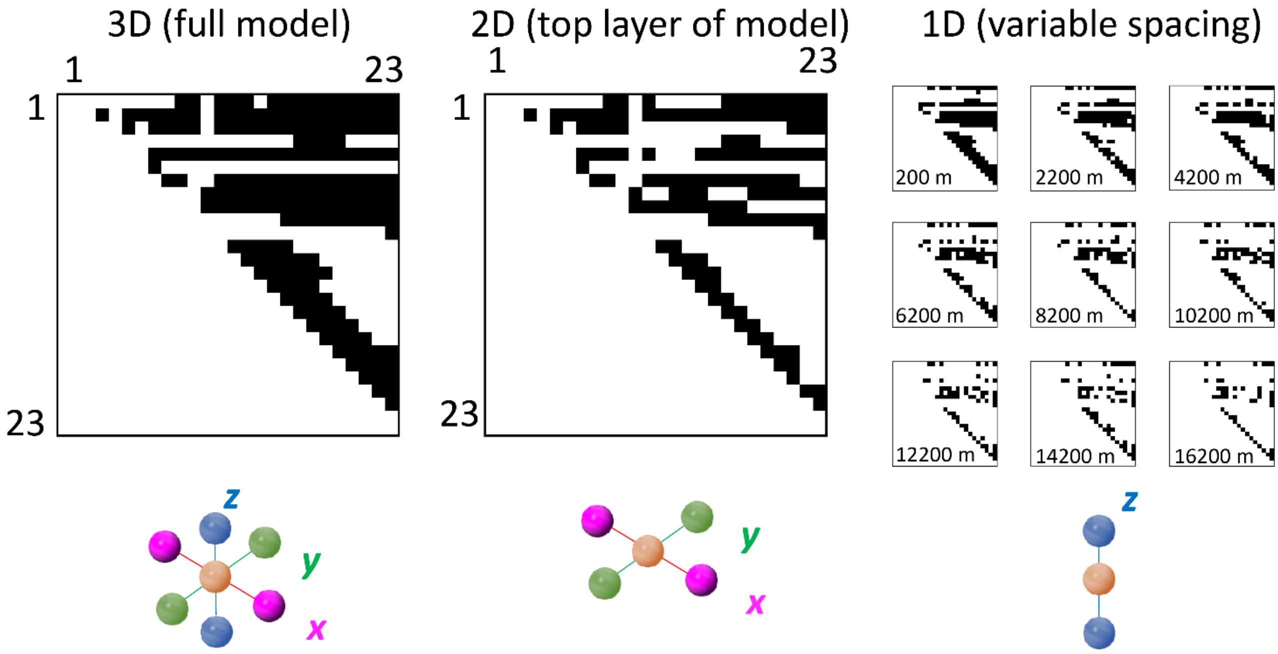

Figure 3. Adjacency Matrices for the full 3D model (left), the top layer of the 3D model (middle) and a series of simulated vertical drill-holes on a grid pattern with increasing distance between points on the grid (right). The 3D model contains the full topological information as defined by the sampling process, whereas the 2D and dense 1D models can already be seen to have found fewer neighbourhood relationships, and by the time we reach a spacing of 17200 m the Adjacency Matrix is quite sparse. The neighbourhood tests for each model are shown schematically below.

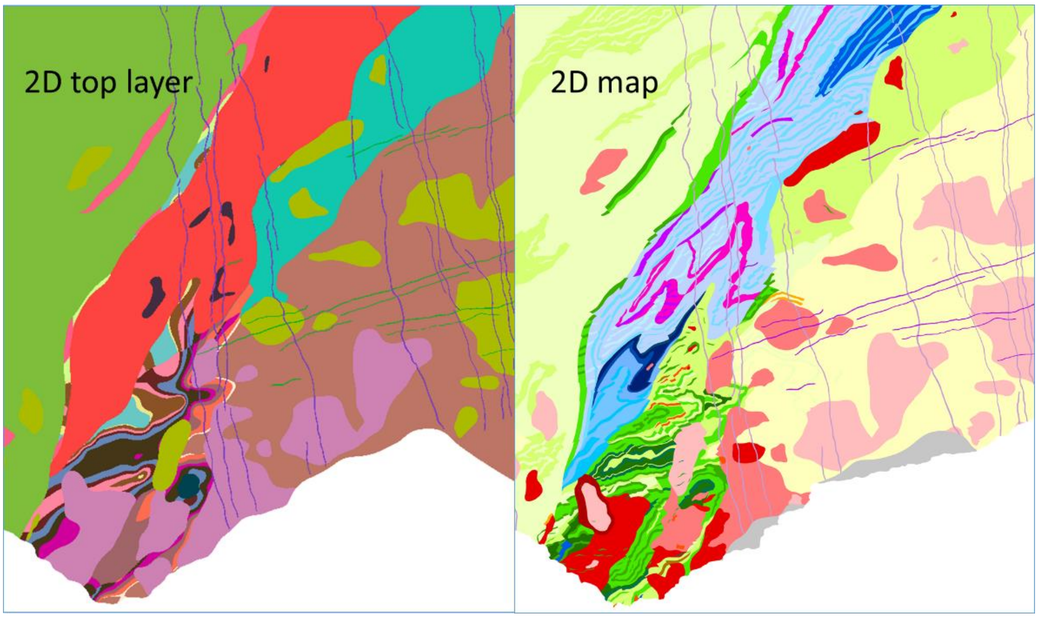

Figure 4 Comparison of the top layer of the 3D geological model (left) and the 2D geological map compiled for the same area, which retains significantly more detail than were able to maintain in the 3D model. Each map is 160 km across.

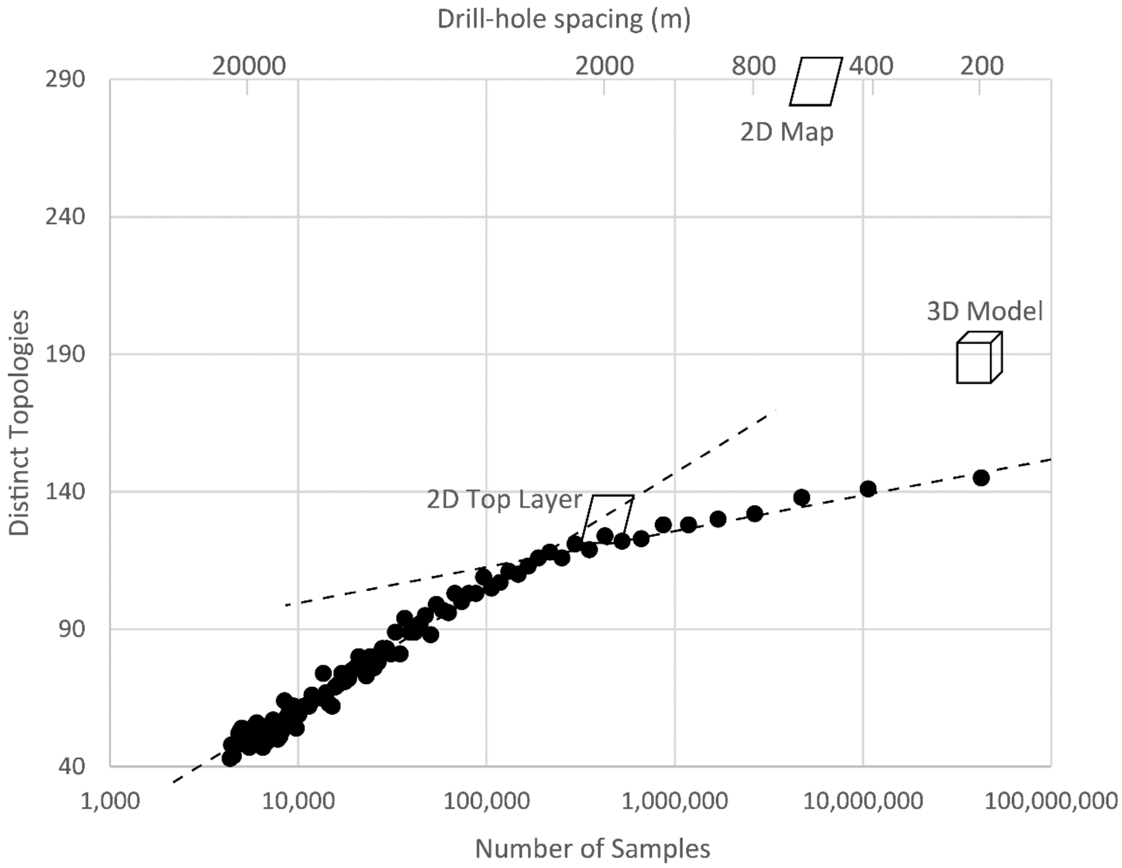

Figure 5 Graph of the number of distinct topological relationships for each model type as a function of the number of elements (voxels or pixels) that are compared within each 1D, 2D or 3D model. For the case of the simulated 1D vertical drill-holes, the number of samples can be considered to be a proxy for the cost of finding out the information, as it is linearly correlated with the summed length of the holes. Block dots show spacing between individual drill-holes, lozenges represent 2D models and the cube represents the full 3D model. Fine lines highlight the change in slope of the relationship for 1D models at 2000 m hole spacing.

The change in slope for the 1D case is at a spacing ten times the voxel size, which may suggest some sort of aliasing, however we have carried out this analysis on other models of real-world and synthetic cases (not shown here) and some models show a break in slope and others don't, and they do not have uniform multiples of voxel size at the break when it does occur. A more likely interpretation of this change in slope is that it reflects changes in the average structural complexity of the model at different length scales, and we will need to undertake further studies for a range of models from different settings to test this hypothesis.

In this study we have only considered simple neighbourhood relationships, ignoring the nature of the relationships (faulted, stratigraphic, intrusive) as well as the surface area of each type of contact. As Thiele et al. (2016 a & b) have shown, for certain modelling and GIS platforms these can also be calculated, and the differences between distinct topologies can be defined by the Jaccard coefficient (Jaccard 1901), which could provide a refined measure for the amount of information in each model.

If real drill-hole data had been available the finer resolution of the logs may have resulted in a much larger number of distinct topologies (equivalent to the increase seen when passing from the top layer of the model to the full 2D map of the same area). In any case this approach uses simple algorithms that allow us to compare the information content in different models that can in turn be used as a metric for the information content of each model, regardless of its dimensionality. A further step, not yet tested, would be to rebuild 3D models for each drill-hole spacing to better understand how well the topology metric correlates with the quality of the resulting 3D models.

The realisation of multiple geological models can be used as one set of inputs for geophysical inversions (Giraud et al., 2017) and the use of topological metrics may be an additional way of comparing inversion outputs to strengthen the geological constraints on the inversion process.

CONCLUSIONS

By reducing the complexity of different representations of geology to simple neighbourhood relationships we are able to compare models of different dimensionality with a view to understanding the inherent information content of each model. In turn, for the case of drill-hole spacing, this provides insights into what density of drilling may be required to provide a sufficient density of information to produce a 3D model that is fit for purpose.

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