Borehole measurements within highly magnetic bodies – corrections of measured magnetic fields and gradients

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SUMMARY

This paper gives explicit expressions for the internal field and gradient components for a layered earth, a layer with gradational susceptibility, a dipping sheet, a sphere and a cylinder, that are exposed to an external field with a uniform gradient. However, the internal field and gradient components cannot be measured directly, as magnetic sensors must be placed within a cavity in the magnetic medium. This modifies the measured field and gradients. For low to moderate susceptibilities, the cavity effect can be calculated assuming that the magnetisation of the surrounding medium is essentially unperturbed by the presence of the cavity. This assumption is unacceptable when the surrounding medium has high susceptibility. I also give expressions that allow the true field and gradient components within a high susceptibility body to be calculated from measurements made in cylindrical cavities, such as boreholes, or in spherical or disc-like cavities.
Key words: Downhole magnetics, magnetic field, magnetic gradient tensor, self-demagnetisation, cavity fields

INTRODUCTION

Forward modelling of magnetic sources is usually based on calculating the external field and gradient components produced by a body of specified shape and magnetic properties. The magnetic properties of the source are generally taken to be homogeneous. This does not necessarily imply, however, that the magnetisation of the source is uniform, because the self-demagnetising field of strongly magnetic sources is generally non-uniform, except for the special case of homogeneous ellipsoidal bodies (Clark *et al*., 1986). The non-uniform resultant internal field (applied field plus self-demagnetising field) produces a non-uniform induced magnetisation in the source, even though its susceptibility is uniform. Bodies with gradational magnetic properties, such as zoned intrusive sills or plugs, zoned alteration systems, or detrital magnetite-bearing sedimentary layers with graded bedding, generate internal magnetic gradients, even though the applied field may be uniform. It is usually assumed when modelling the external and internal effects of a magnetic source that the applied field is the regional geomagnetic field, which can be taken as uniform. When making gradient measurements, however, it is important to understand the effect of the local magnetic medium on the background gradient. Furthermore neighboring strongly magnetic sources can perturb the regional field and impose gradients on the target source. For this reason, it is useful to study the effects of the local magnetic environment on imposed gradients, as well on the ambient field.

Measurement of magnetic gradients, especially measuring the full gradient tensor, has many advantages (Pedersen and Rasmussen, 1990; Schmidt and Clark, 2006) over conventional magnetic surveys, particularly when spatial coverage is sparse. This is especially true for borehole magnetic measurements, which provide a single string of data. Measuring TMI downhole provides a profile of only a single magnetic component, whereas measuring the field vector provides three components at each measurement location, but these tend to be very noisy due to orientation uncertainties. Measuring the full gradient tensor provides five independent components at each observation point, with much less sensitivity to orientation errors, which allows better targeting of nearby sources (Pedersen and Rasmussen, 1990; Schmidt and Clark, 2006; Clark, 2012, 2013). Borehole magnetic measurements, in particular, can be very useful for targeting subsurface sources (Levanto, 1963; Bosum *et al*., 1988; Hoschke, 1991; Hillan *et al*., 2012) and can provide useful information about the source magnetisation, especially when the borehole intersects the magnetic source (Clark, 2014). It is therefore important to (i) model the internal field of magnetic sources correctly, and (ii) calculate the field and gradients that are actually measured in a borehole, or other cavity, inside a magnetic body so that borehole magnetic data can be interpreted correctly. Leslie *et al*. (2015) describe a downhole tool capable of measuring the full magnetic gradient tensor that is being developed to assist mineral exploration.

Within a medium with magnetisation $\mathbf{M}$, the magnetic field intensity $\mathbf{H}$, and the flux density $\mathbf{B}$ are related by $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$, where μ_0 is the permeability of free space. Assuming no remanent magnetisation and an induced magnetisation $\mathbf{M} = \chi\mathbf{H}$ that is linear in the field intensity, then $\mathbf{B} = \mu_0(1+\chi)\mathbf{H} = \mu_0\mu\mathrm{H}$, where χ is the susceptibility and $\mu = 1 + \chi$ is the relative permeability of the medium. This paper examines how fields and field gradients within a highly magnetic medium, due to external magnetic sources, are affected by the shape and permeability of the magnetic body within which measurements are made. Figure 1 illustrates the simple relationship between the measured magnetic field in a borehole that intersects an initially uniformly magnetised body (e.g. a homogeneous ellipsoid in a uniform external field) and the unperturbed internal field and magnetisation of the body. It will be shown in this paper that the effects of self-demagnetisation on gradients are somewhat more complicated than its effects on field components.

The field vectors and their gradients cannot be measured directly within a permeable medium; rather, the fields and gradients in the medium must be inferred from measurements made by instruments situated in a cavity, such as a borehole, within the medium. This paper shows how measurements made within a spherical or disc-like cavity or within a borehole, approximated as an infinitely long cylindrical cavity, can be corrected to infer the fields and gradients within the surrounding magnetic medium.

MAGNETIC SCALAR POTENTIAL OF AN APPLIED FIELD WITH A UNIFORM GRADIENT

The magnetic scalar potential Ω_0 is only defined to within an arbitrary additive constant, so for simplicity Ω_0 can be set to zero at the origin of co-ordinates. Then, in terms of Cartesian co-ordinates (x, y, z), the magnetic scalar potential associated with an applied field with a superimposed uniform gradient tensor **G** is:

$$\begin{aligned}\Omega_0 = &-(\overline{H}_0)_x x - (\overline{H}_0)_y y - (\overline{H}_0)_z z + \frac{G_{xx}}{6}(y^2+z^2-2x^2) - G_{xy}xy - G_{xz}xz \\ &+\frac{G_{yy}}{6}(x^2+z^2-2y^2) - G_{yz}yz - \frac{(G_{xx}+G_{yy})}{6}(x^2+y^2-2z^2),\end{aligned} \quad (1)$$

where $\overline{\mathbf{H}}_0$ is the field, averaged over all space, which is also equal to the field vector at the origin, and the components of the gradient tensor **G** are $G_{ij} = \partial H_i/\partial j = -\partial^2\Omega/\partial i\partial j$ ($i,j = x, y, z$).

The matrix of Cartesian elements of the uniform gradient tensor is

$$\mathbf{G} = \nabla\mathbf{H} = -\nabla\nabla\Omega_0 = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{xy} & G_{yy} & G_{yz} \\ G_{xz} & G_{yz} & G_{zz} \end{bmatrix} = \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{xy} & G_{yy} & G_{yz} \\ G_{xz} & G_{yz} & -(G_{xx}+G_{yy}) \end{bmatrix}. \quad (2)$$

The mathematical form of equation (1) ensures that the corresponding gradient tensor, given by (2), is symmetric and traceless, as required by Maxwell's equations for the magnetostatic field, in the absence of conduction currents. This implies that there are only five independent gradient tensor components, e.g. G_{xx}, G_{xy}, G_{xz}, G_{yy}, G_{yz}. In (1) the functions of x, y, z associated with each tensor element are harmonic and homogeneous functions of order two. Note that the magnetic gradient tensor is defined here, and throughout this paper, as the vector gradient of the magnetic field intensity **H**, rather the gradient of the flux density **B**, and is expressed in units of A/m². In a linear magnetic medium with no remanence (or a uniform remanence) the gradient of the flux density is $\nabla\mathbf{B} = \mu\nabla\mathbf{H} = \mu\mathbf{G}$. Gradiometer measurements are made in a locally non-magnetic environment (air, water, oil, outer space), for which $\nabla\mathbf{B} = \mu_0\mathbf{G}$, so interconversion between *measured* gradients of **B** and **H** is straightforward (G_{ij} = 1 A/m² $\Rightarrow (\nabla\mathbf{B})_{ij} = 4\pi \times 10^{-7}$ T/m = 1257 nT/m) .

As a function of spatial position, the field corresponding to the potential in (1), is

$$\begin{aligned}\mathbf{H}(x,y,z) = -\nabla\Omega_0 = \overline{\mathbf{H}}_\mathbf{0} + \mathbf{G.r} = &[(H_0)_x + G_{xx}x + G_{xy}y + G_{xz}z]\hat{\mathbf{x}} + [(H_0)_y + G_{xy}x + G_{yy}y + G_{yz}z]\hat{\mathbf{y}} \\ &+[(H_0)_z + G_{xz}x + G_{yz}y + G_{zz}z]\hat{\mathbf{z}}.\end{aligned} \quad (3)$$

where $\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}$ are unit vectors along the co-ordinate axes.

With respect to spherical polar co-ordinates (r,θ,φ), Ω_0 is:

$$\begin{aligned}\Omega_0 = &-(H_0)_x r\cos\varphi\sin\theta - (H_0)_y r\sin\varphi\sin\theta - (H_0)_z r\cos\theta + \frac{G_{xx}}{6}r^2(1-3\cos^2\varphi\sin^2\theta) - \frac{G_{xy}}{2}r^2\sin 2\varphi\sin^2\theta \\ &-\frac{G_{xz}}{2}r^2\cos\varphi\sin 2\theta + \frac{G_{yy}}{6}r^2(1-3\sin^2\varphi\sin^2\theta) - \frac{G_{yz}}{2}r^2\sin\varphi\sin 2\theta - \frac{(G_{xx}+G_{yy})}{6}r^2(1-3\cos^2\theta),\end{aligned} \quad (4)$$

and with respect to cylindrical co-ordinates (ρ, ϕ, z)

$$\begin{aligned}\Omega_0 = &-(H_0)_x\rho\cos\phi - (H_0)_y\rho\sin\phi - (H_0)_z z - \frac{G_{xx}}{4}\rho^2\cos 2\phi - \frac{G_{xy}}{2}\rho^2\sin 2\phi \\ &-G_{xz}\rho z\cos\phi + \frac{G_{yy}}{4}\rho^2\cos 2\phi - G_{yz}\rho z\sin\phi - \frac{(G_{xx}+G_{yy})}{6}(\rho^2-2z^2).\end{aligned} \quad (5)$$

INFINITE SHEET WITHIN A PERMEABLE MEDIUM – GEOMAGNETIC FIELD WITH A UNIFORM GRADIENT

Consider the case of a magnetic subsurface of permeability μ_1, from the surface (or the top of the magnetic basement) at $z = 0$ to a depth $z = z_3$, with an embedded horizontal layer of different permeability μ_2 over the depth range $0 \le z_1 \le z \le z_2 \le z_3$. The boundary conditions imply that the vertical component of **B** and the horizontal components of **H** are continuous through the horizontal interfaces. This implies that the vertical component of **H** undergoes step changes at each interface between layers of different permeability, but gradient tensor components that only involve x or y are unaffected by the horizontally layered structure. It turns out that the diagonal gradient tensor components in all zones are unperturbed by the layering. However a geomagnetic gradient tensor, or a gradient tensor produced by a neighbouring source, with nonzero xz or yz components is perturbed by the magnetic layering.

For a surface geomagnetic field $\mathbf{H} = (X, Y, Z)$ with a superimposed uniform gradient tensor $\mathbf{G}_0$, the corresponding fields within the i^{th} depth zone (i = 1,2,3,4) can be calculated by noting that within each layer of relative permeability μ_i the gradient tensor components in the permeable medium are related to the gradient tensor above the surface $\mathbf{G}_0$ by:

$$\mathbf{G}_i = \begin{bmatrix} (G_0)_{xx} & (G_0)_{xy} & (G_0)_{xz}/\mu_i \\ (G_0)_{xy} & (G_0)_{yy} & (G_0)_{yz}/\mu_i \\ (G_0)_{xz}/\mu_i & (G_0)_{yz}/\mu_i & (G_0)_{zz} \end{bmatrix}, \tag{6}$$

and by tracking the step changes in vertical field across each interface, which are given by:

$$(\Delta H_1)_z = (H_1)_z - Z = Z\left[\frac{1}{\mu_1} - 1\right]; \quad (\Delta H_i)_z = (H_i)_z - (H_{i-1})_z = (H_{i-1})_z\left[\frac{\mu_{i-1}}{\mu_i} - 1\right]. \tag{7}$$

Starting at the surface or just above the top of the first nonmagnetic layer, the fields within each layer are extrapolated to the next interface by substituting the gradient components in (6) into (3) (where in this case z represents the increment in depth within that layer), to the field just below the previous interface, which in turn is determined by adding the step change given by (7) to the field just above the previous interface.

These results may also be applied to the relatively common case of a dipping magnetic sheet of relative permeability $\mu = 1+\chi$ in a non-magnetic half-space, provided the gradient tensor components are expressed in terms of Cartesian axes with x down-dip, y along strike, and z normal to the plane of the sheet. Then, for points within the sheet that are several sheet-thicknesses away from the top and bottom edges so that edge effects are negligible, the gradient of $\mathbf{H}$ is continuous across the boundary of the sheet for all components except for G_{xz} and G_{yz}, which are decreased by a factor $1/(1+\chi)$ within the sheet.

The same principles that apply to infinite sheets may also be applied to measurements made in a thin disc-like or slot-like cavity (χ = 0) within a medium of permeability μ. In this case the gradient tensor $\mathbf{G}$ in the surrounding medium is related to the measured gradient tensor $\mathbf{G}_{cav}$ by:

$$\mathbf{G} = \begin{bmatrix} (G_{cav})_{xx} & (G_{cav})_{xy} & (G_{cav})_{xz}/\mu \\ (G_{cav})_{xy} & (G_{cav})_{yy} & (G_{cav})_{yz}/\mu \\ (G_{cav})_{xz}/\mu & (G_{cav})_{yz}/\mu & (G_{cav})_{zz} \end{bmatrix}, \tag{8}$$

where the x and y axes lie in the plane of the disc and the z axis is normal to the disc. Thus the gradient tensor elements in the surrounding medium can be inferred from the measured gradient tensor in the cavity, if the susceptibility of the medium is known. The xz and yz components in the cavity must be divided by the relative permeability; the other components are unchanged by the presence of the cavity. For a disc-like cavity the external $\mathbf{H}$ field components in the plane of the disc are unchanged within the cavity, but the cavity field normal to the disc must be divided by μ to obtain the normal component of the external field.

SPHERICAL BODY OR CAVITY WITHIN A PERMEABLE MEDIUM

Consider a spherical homogeneous magnetisable body of radius a, with relative permeability μ_1, within an effectively infinite magnetisable medium with relative permeability μ_2. We assume that there is no permanent magnetisation in either medium. If the unperturbed applied field has a uniform gradient $\mathbf{G}_0$, we may conveniently employ spherical polar co-ordinates with origin at the centre of the sphere and the polar (z) axis chosen to be parallel to the unperturbed field at the position of the sphere. The unperturbed scalar potential, far from the cavity, is then given by (4) with $(H_0)_x = (H_0)_y = 0$.

The boundary conditions on the scalar potential are: (i) continuity of the potential across the spherical boundary, (ii) continuity of the normal (i.e. radial) component of the magnetic flux density $\mathbf{B} = \mu\mu_0\mathbf{H}$ across the spherical boundary, (iii) regularity of the potential inside the sphere, and (iv) convergence of the external potential to expression (4) as $r \to \infty$. Solving Laplace's equation with these boundary conditions and calculating the fields from the scalar potentials yields:

$$\mathbf{H}(r < a) = \frac{3\mu_1}{2\mu_1 + \mu_2}\overline{\mathbf{H}}_0 + \frac{5\mu_1}{3\mu_1 + 2\mu_2}\mathbf{G}_0.\mathbf{r}, \tag{9}$$

for the internal field. Equation (9) shows that the internal field also has a uniform gradient. The field at the centre of the sphere is multiplied by a factor of $3\mu_1/(2\mu_1 + \mu_2)$ compared to the unperturbed field at the same location. For the case of a permeable sphere, with $\mu_2 = 1+\chi$, in a nonmagnetic medium ($\mu_1 = 1$) the internal field at the centre is reduced with respect to the applied field, due to self-demagnetisation:

$$\mathbf{H}(r = 0; \mu_1 = 1, \mu_2 = 1+\chi) = \frac{\chi}{1+\chi/3}\overline{\mathbf{H}}_0. \tag{10}$$

This is consistent with the well-known formula for the induced magnetisation in a uniform applied field, of a permeable sphere in a vacuum, $\mathbf{M} = \chi\mathbf{H} = \chi'\mathbf{H}_0$, for which the effective susceptibility, corrected for self-demagnetisation, is $\chi' = \chi/(1+\chi/3)$.
On the other hand, the field inside a spherical cavity within a permeable medium ($\mu_1 = 1+\chi$, $\mu_2 = 1$) is given by:

$$\mathbf{H}(r=0;\mu_1 = 1+\chi, \mu_2 = 1) = \frac{1+\chi}{1+2\chi/3}\overline{\mathbf{H}}_0. \tag{11}$$

Thus the field at the centre of the spherical cavity is amplified by a factor of $(1+\chi)/(1+2\chi/3)$ with respect to the unperturbed applied field at that point. For relatively small susceptibilities ($\chi << 1$), this amplification factor is, to a good approximation, equal to $(1+\chi/3)$, which is the cavity field calculated by assuming a uniform magnetisation equal to $\chi\mathbf{H}_0$ in the external medium. For $\chi \geq 0.5$, there is a significant difference between the exact calculation and the uniform-magnetisation approximation, due to the perturbation of the applied field by the presence of the cavity. The external field of the sphere is equivalent to that produced by point dipole plus point quadrupole sources at the centre of the sphere. The dipole term is produced by the average applied field, the quadrupole term arises from the applied gradient. The magnetic moment of the equivalent dipole is:

$$\mathbf{m} = \frac{3(\mu_2 - \mu_1)\mathbf{H}_0}{(2\mu_1 + \mu_2)} = \frac{(\chi_2 - \chi_1)\mathbf{H}_0}{1+(2\chi_1+\chi_2)/3}. \tag{12}$$

Note that, if $\chi_1 > \chi_2$ (in particular, for a cavity in a magnetic medium), then the dipole moment is directed opposite to the applied field. The total magnetic moment arises from both the magnetisation of the material in the sphere and from the distribution of magnetic poles induced on the inner surface of the spherical cavity that has been "carved out" of the surrounding permeable medium in order to accommodate the sphere.

For a spherical cavity within an infinite permeable medium ($\mu_1 = 1+\chi$; $\mu_2 = 1$), the internal gradient tensor is amplified with respect to the applied gradient:

$$\mathbf{G}(r<a;\mu_1 = 1+\chi, \mu_2 = 1) = \frac{1+\chi}{1+3\chi/5}\mathbf{G}_0. \tag{13}$$

For a spherical permeable body within an infinite nonmagnetic medium ($\mu_1 = 1$, $\mu_2 = 1+\chi$), the internal gradient tensor is somewhat shielded with respect to the applied gradient:

$$\mathbf{G}(r<a;\mu_1 = 1, \mu_2 = 1+\chi) = \frac{1}{1+2\chi/5}\mathbf{G}_0. \tag{14}$$

Thus the shielding factor for first-order gradients inside a magnetisable sphere, $1/(1+2\chi/5)$, differs from the shielding factor for the applied field, which is $1/(1+\chi/3)$.

CYLINDRICAL BODY OR CAVITY WITHIN A PERMEABLE MEDIUM

Consider a magnetisable body in the form of a very long cylinder of radius a, with relative permeability μ_2, within an effectively infinite magnetisable medium with relative permeability μ_1. We assume that there is no permanent magnetisation in either medium. If the unperturbed applied field has a uniform gradient, we may employ cylindrical polar co-ordinates with origin on the axis of the cylinder, which coincides with the z axis. The potentials and fields in this case are independent of z. Continuity of the field component H_z across the walls of the cylinder ensures that this component is unperturbed by the presence of the cylinder. Thus we only need to consider the applied field components that lie in the plane perpendicular to the cylinder axis. Let the polar axis, for which $\phi = 0$, coincide with the unperturbed applied field direction at the origin. The unperturbed scalar potential, far from the cavity, is then given by (5) with $(H_0)_x = (H_0)_y = 0$.

Solving Laplace's equation with the appropriate boundary conditions and calculating the fields from the scalar potentials yields:

$$\mathbf{H}(\rho < a) = \frac{2\mu_1}{\mu_1+\mu_2}\overline{\mathbf{H}}_0 + \mathbf{G}'.\mathbf{r}, \tag{15}$$

for the internal field, where the modified gradient tensor inside the cylinder is:

$$\mathbf{G}' = \begin{bmatrix} \dfrac{(3\mu_1+\mu_2)G_{xx} + (\mu_2-\mu_1)G_{yy}}{2(\mu_1+\mu_2)} & \dfrac{2\mu_1}{(\mu_1+\mu_2)}G_{xy} & \dfrac{2\mu_1}{(\mu_1+\mu_2)}G_{xz} \\ \dfrac{2\mu_1 G_{xy}}{(\mu_1+\mu_2)} & \dfrac{(3\mu_1+\mu_2)G_{yy} + (\mu_2-\mu_1)G_{xx}}{2(\mu_1+\mu_2)} & \dfrac{2\mu_1}{(\mu_1+\mu_2)}G_{yz} \\ \dfrac{2\mu_1}{(\mu_1+\mu_2)}G_{xz} & \dfrac{2\mu_1}{(\mu_1+\mu_2)}G_{yz} & G_{zz} \end{bmatrix}. \tag{16}$$

Equations (15) and (16) show that the internal field also has a uniform gradient. The axial field and its axial gradient are unperturbed by the presence of the cylinder. The transverse field on the axis of the cylinder is multiplied by a factor of $2\mu_1/(\mu_1+\mu_2)$ compared to the unperturbed transverse field at the same location. For the case of a permeable cylinder, with $\mu_2 = 1+\chi$, in a nonmagnetic medium ($\mu_1 = 1$) the internal transverse field on the axis is reduced with respect to the applied transverse field, due to self-demagnetisation:

$$H_{x,y}(\rho=0,z;\mu_1=1,\mu_2=1+\chi)=\frac{\chi}{1+\chi/2}(\mathbf{H}_0)_{x,y}(z). \tag{17}$$

This is consistent with the well-known formula for the magnetisation induced by a uniform applied transverse field $\mathbf{H}_0$ in a permeable cylinder in a vacuum, $\mathbf{M} = \chi\mathbf{H} = \chi'\mathbf{H}_0$, for which the effective susceptibility, corrected for self-demagnetisation, is $\chi' = \chi/(1+\chi/2)$.

The anomalous external potential and field are identical to those produced by an infinite line of dipoles along the cylinder axis, aligned with the transverse component of the applied field, plus a line of quadrupoles that are associated with the applied gradient, and a line of poles associated with the axially symmetric portion of the gradient tensor ($G_{xx} = G_{yy} = -G_{zz}/2$). The latter gradient configuration produces an axially symmetric distribution of magnetic poles, associated with the radial magnetisation contrast across the walls of the cylinder, that lines the surface of the cylinder and is evenly distributed along the length of the cylinder. Such a pole distribution produces no internal field. Externally, the anomalous potential corresponds to that of a line of poles uniformly distributed along the cylinder axis. The potential associated with such a line of poles has a logarithmic dependence on radial distance, but is independent of z. There is a seeming paradox due to an apparent excess of poles on the outer surface of the cylinder. This can be explained by noting that the "missing poles" (which are required to ensure that the total pole strength produced by the magnetisation distribution sums to zero) are to be found at the ends of the cylinder, which are assumed to be sufficiently distant that their fields can be neglected.

The dipole moment per unit length of the cylinder is

$$M'A=2\left(\frac{\mu_2-\mu_1}{\mu_1+\mu_2}\right)H_0A=2\left(\frac{\chi_2-\chi_1}{\mu_1+\mu_2}\right)H_0A, \tag{18}$$

where $A = \pi a^2$ is the cross-section area of the cylinder and M' is its effective transverse magnetisation. This differs from the internal transverse magnetisation $M = (\mu_2-1)H_2$ of the cylinder, because the magnetisation contrast across the walls of the cylinder produces a surface density of magnetic charge that also contributes to the anomalous potential and field. Note that the effective magnetisation of the cylinder is not simply proportional to the susceptibility contrast between the external and internal media. However, if the external medium is non-magnetic ($\mu_1 = 1$), and the internal permeability is $\mu_2 = 1+\chi$, then the internal field is $H_2 = H_0/(1+\chi/2)$ and the magnetisation of the cylinder is $\chi H_0/(1+\chi/2) = \chi' H_0$, where χ' is the demagnetisation-corrected effective susceptibility for an infinite cylinder in a non-magnetic medium. If the susceptibility of the cylinder is less than that of the surrounding medium, the equivalent line of dipole moments is oriented opposite to the applied field.

On the other hand, the transverse field on the axis of a cylindrical cavity within a permeable medium ($\mu_1 = 1+\chi$, $\mu_2 = 1$) is given by:

$$H_{x,y}(\rho=0,z;\mu_1=1+\chi,\mu_2=1)=\frac{2(1+\chi)\mu_1}{2+\chi}=\frac{1+\chi}{1+\chi/2}(H_0)_{x,y}. \tag{19}$$

Thus the field along the axis of the cylindrical cavity is amplified by a factor of $(1+\chi)/(1+\chi/2)$ with respect to the applied field. For relatively small susceptibilities ($\chi << 1$), this amplification factor is, to a good approximation, equal to $(1+\chi/2)$, which is the cavity field calculated by assuming a uniform magnetisation equal to $\chi\mathbf{H}_0$ in the external medium. For $\chi \geq 0.5$, there is a significant difference between the exact calculation and the uniform-magnetisation approximation, due to the perturbation of the applied field by the presence of the cavity.

Applying the tracelessness property to (16), the gradient tensor measured within a cylindrical cavity within a permeable medium is:

$$\mathbf{G}'(\rho<a;\mu_1=1+\chi,\mu_2=1)=\begin{bmatrix}\dfrac{(1+\chi)G_{xx}-\chi G_{zz}/4}{(1+\chi/2)} & \dfrac{(1+\chi)}{(1+\chi/2)}G_{xy} & \dfrac{(1+\chi)}{(1+\chi/2)}G_{xz}\\ \dfrac{(1+\chi)}{(1+\chi/2)}G_{xy} & \dfrac{(1+\chi)G_{yy}-\chi G_{zz}/4}{(1+\chi/2)} & \dfrac{(1+\chi)}{(1+\chi/2)}G_{yz}\\ \dfrac{(1+\chi)}{(1+\chi/2)}G_{xz} & \dfrac{(1+\chi)}{(1+\chi/2)}G_{yz} & G_{zz}\end{bmatrix}. \tag{20}$$

Equation (16) shows that the xy, xz, and yz components of the gradient tensor inside a permeable cylinder ($\mu_2 = 1+\chi$) within a non-permeable medium ($\mu_1 = 1$) are attenuated by self-demagnetisation, with a shielding factor of $1/(1+\chi/2)$, which is identical to the shielding factor for H_x and H_y. On the other hand, equation (20) shows that for a cylindrical cavity in a permeable medium, these components of the applied gradient within the surrounding medium are amplified by a factor of $(1+\chi)/(1+\chi/2)$. Thus the shielding/amplification factors for the off-diagonal components of the applied gradient tensor are the same as for the transverse components of the field. The zz component of the gradient is unaffected by the presence of the cylindrical body or cavity. However the responses of the other diagonal components of the gradient tensor are more complicated than a simple shielding/amplification.

Manipulation of equation (20) gives the following expressions for the true gradient tensor $\mathbf{G}_0$ in the surrounding medium, corrected for the effects of the borehole cavity, expressed in terms of the tensor elements measured in the borehole:

$$\begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{xy} & G_{yy} & G_{yz} \\ G_{xz} & G_{yz} & G_{zz} \end{bmatrix} = \begin{bmatrix} \dfrac{(1+\chi/2)G'_{xx} + \chi G'_{zz}/4}{(1+\chi)} & \dfrac{(1+\chi/2)}{(1+\chi)}G'_{xy} & \dfrac{(1+\chi/2)}{(1+\chi)}G'_{xz} \\ \dfrac{(1+\chi/2)}{(1+\chi)}G'_{xy} & \dfrac{(1+\chi/2)G'_{yy} + \chi G'_{zz}/4}{(1+\chi)} & \dfrac{(1+\chi/2)}{(1+\chi)}G'_{yz} \\ \dfrac{(1+\chi/2)}{(1+\chi)}G'_{xz} & \dfrac{(1+\chi/2)}{(1+\chi)}G'_{yz} & G'_{zz} \end{bmatrix}. \tag{21}$$

Within a medium that has relatively low susceptibility, such as most sedimentary rock units, the required corrections to the measured gradient tensor elements are approximately:

$$\left.\begin{array}{r} G_{ii} \approx (1-\chi/2)G'_{ii} + \chi G'_{zz}/4, \quad (i = x, y) \\ G_{ij} \approx (1-\chi/2)G'_{ij}, \quad (i \neq j) \\ G_{zz} = G'_{zz} \end{array}\right\}, \ (\chi << 1). \tag{22}$$

BOREHOLE WITHIN A REMANENTLY MAGNETISED PERMEABLE MEDIUM

Except near the boundaries of a magnetic rock unit, a borehole can be modelled as a long, effectively infinite, cylindrical cavity. Figure 2 illustrates the relationship between the measured magnetic field in a vertical borehole that intersects a horizontal layer and the unperturbed field and magnetisation in the layer, in the simple case of homogeneous properties and a uniform external field. If the permeable medium (relative permeability μ) surrounding the borehole has a uniform remanent magnetisation $\mathbf{R}$, the remanence produces an additional uniform field within the borehole. The axially directed component of $\mathbf{R}$ has no effect. The transverse field in the cavity produced by the remanence of the surrounding medium is proportional to and parallel to the transverse component $R_\perp$ of $\mathbf{R}$. Solving the elementary boundary value problem for this case shows that the transverse anomalous field vector in the borehole is:

$$\Delta\mathbf{H}_\perp = \frac{\chi\mathbf{H}_\perp + \mathbf{R}_\perp}{\mu+1} = \frac{\mathbf{M}_\perp}{\mu+1} = \frac{\mathbf{M}_\perp}{2+\chi}, \tag{23}$$

where $\mathbf{M}_\perp$ is the transverse component of the vector sum of the induced and remanent magnetisations. The component of magnetisation parallel to the borehole can be estimated from the fairly abrupt step-like change in the axial component of $\mathbf{H}$ that occurs over a distance of several borehole diameters as the borehole sensor passes into the sheet (see Figure 3). In order to estimate the magnetisation vector, the orientation of the sheet must be known, so that the direction of the self-demagnetising field (normal to the sheet) can be ascertained.

If the susceptibility of the formation that is intersected by the borehole is known, by borehole logging or measurements on samples of the formation, then the total magnetisation and remanent magnetisation can be estimated from measurements of the anomalous magnetic field vector in the borehole:

$$\mathbf{M}_\perp = (\mu+1)\Delta\mathbf{H}_\perp = (2+\chi)\Delta\mathbf{H}_\perp; \ \ \mathbf{R}_\perp = \mathbf{M}_\perp - \chi\mathbf{H}_\perp. \tag{24}$$

If the susceptibility of the sheet is known to be reasonably low (χ << 1), (μ+1) can be set equal to 2 and the magnetisation can be inferred directly from the borehole field measurements. Otherwise, reliable estimation of the transverse components of magnetisation requires an independent measurement of the susceptibility of the sheet.

EFFECTS OF ZONED SUSCEPTIBILITIES ON BOREHOLE FIELD AND GRADIENT MEASUREMENTS

A rock unit with zoned susceptibility has non-uniform magnetisation and produces internal gradients, even when the applied field is uniform. Examples of such units include thick intrusive sills that have undergone crystal settling, layered mafic/ultramafic intrusions, alteration zones, and detrital magnetite-bearing sandstones with graded bedding. For simplicity consider a horizontal unit of thickness T, in which the susceptibility varies linearly from χ_1 to χ_2 between its upper and lower boundaries at z_1 and z_2. The vertical component of $\mathbf{B}$ is continuous across the upper and lower boundaries of the unit and is constant within the magnetic unit. This implies that $H_z = B_z/\mu_0(1+\chi)$ varies systematically with depth. For a vertical borehole well within the magnetic unit (i.e. several borehole diameters from either interface) H_z in the borehole is equal to H_z in the adjacent rock, so in the borehole the measured vertical component of the flux density, $B_z' = \mu_0 H_z'$, varies systematically with depth. If there is no imposed gradient, i.e. the external field is uniform, the measured vertical component of the field and its vertical gradient are given by:

$$\begin{aligned} B'_z &= \mu_0 H'_z = \mu_0 H_z = \frac{B_z}{1+\chi(z)} = \frac{B_z}{1+\chi_1+(\chi_2-\chi_1)(z-z_1)/T} = \frac{B_z}{1+\chi_1+(\partial\chi/\partial z)(z-z_1)}, \\ B'_{zz} &= \frac{-B_z(\partial\chi/\partial z)}{[1+\chi(z)]^2} = \frac{-B_z(\partial\chi/\partial z)}{[1+\chi_1+(\partial\chi/\partial z)(z-z_1)]^2}. \end{aligned} \tag{25}$$

Thus the gradient in susceptibility produces a field gradient inside the borehole. If the external vertical field has a superimposed uniform vertical gradient B_{zz} then $B_z(z) = B_z(z_1) + (z - z_1)B_{zz}$ along a vertical profile through the sheet. In a vertical borehole well within the sheet the measured vertical field and its vertical gradient are given by:

$$\begin{aligned} B_z'(z) &= \frac{B_z(z)}{1+\chi} = \frac{B_z(z_1) + B_{zz}(z-z_1)}{1+\chi} = \frac{B_z(z_1) + B_{zz}(z-z_1)}{1+\chi_1 + (\partial\chi/\partial z)(z-z_1)}, \\ B_{zz}'(z) &= \frac{B_{zz}}{1+\chi(z)} - \frac{B_z(z)(\partial\chi/\partial z)}{[1+\chi(z)]^2} = \frac{B_{zz}}{1+\chi(z)} - \frac{[B_z(z_1) + B_{zz}(z-z_1)](\partial\chi/\partial z)}{(1+\chi)^2}. \end{aligned} \tag{26}$$

Thus the external gradient is attenuated by factor of $1/[1+\chi(z)]$, which is the standard shielding factor for this geometry, *and* the gradient in susceptibility contributes an additional field gradient inside the borehole. The latter term can be substantial, when there is a significant gradient in the susceptibility. Systematic changes in remanent magnetisation with depth produce similar effects and can be calculated using these formulas by converting the magnetisation $M_z(z)$ into an equivalent susceptibility equal to $M_z(z)/H_z(z)$.

If the susceptibility of the intersected rock units is known, or is logged during drilling, simple corrections can be made to measured field and gradient data if either (i) the susceptibility is fairly uniform throughout a given stratigraphic interval (even if it is fairly high), or (ii) the susceptibility varies gradually and is measured through the stratigraphic interval. On the other hand, large susceptibility variations over short stratigraphic intervals will introduce substantial noise into magnetic measurements. Similarly, measurements within a strongly magnetic rock unit that has highly heterogeneous magnetic properties will be afflicted by substantial geological noise, which could mask the signatures of nearby target sources.

CONCLUSIONS

The internal magnetic field of a magnetised body is affected by self-demagnetisation, which in general depends both on its shape and on the susceptibilities of the body and its host rock. The effects of self-demagnetisation on the internal gradient tensor differ from the effects on the internal field. Measured magnetic field vectors and gradient tensor components in a borehole or other cavity differ from the corresponding quantities in the surrounding rock. Inferring the external field and its gradient in the adjacent material from measurements that are necessarily made in a cavity within the magnetised rock, requires knowledge of, or assumptions about, the geometry of the magnetic body within which the measurements are made. Furthermore the susceptibility of the rock surrounding the borehole affects the correction that is required to infer the field and gradients in the surrounding rock. A rock unit with gradational magnetic properties has internal magnetic gradients (in addition to imposed gradients from external sources), which are detectable within a borehole that intersects the unit. This paper gives expressions that relate measured fields and gradients to the corresponding quantities in the surrounding rock, which should assist interpretation of borehole data, in particular, in highly magnetic environments.

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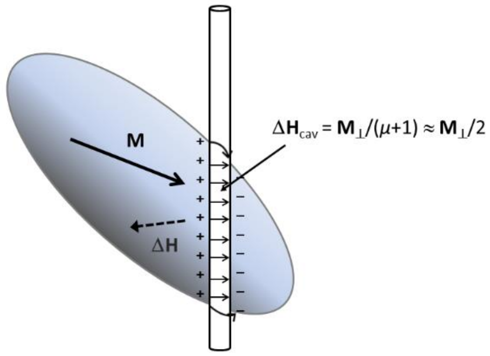

Figure 1: Relationship between the uniform unperturbed internal magnetic field for a homogeneous ellipsoidal body and the field measured within a borehole that intersects the body.

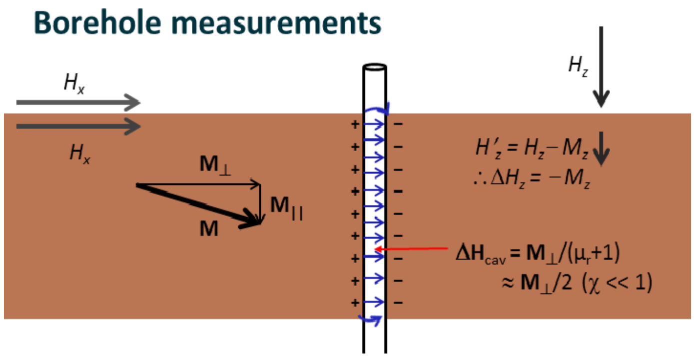

Figure 2: Relationships between the unperturbed external and internal magnetic fields for a uniformly magnetised horizontal layer and for the field measured within a borehole that intersects the layer.

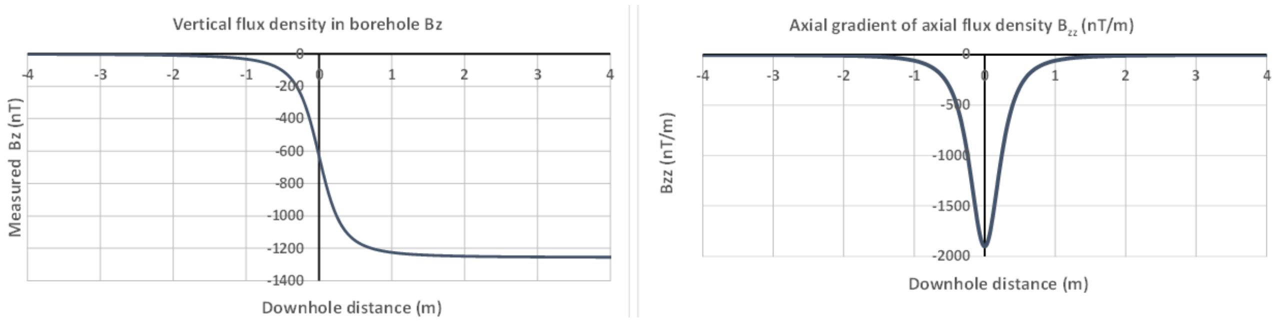

Figure 3: Anomalous vertical field and its vertical gradient along the axis of a 66 cm (26 inch) diameter borehole as it passes from a non-magnetic formation into a thick horizontal sheet with a vertical magnetisation of 1 A/m. Laterally away from the borehole the H field undergoes a sharp step change across the upper surface of the magnetic sheet. However the borehole produces a cylindrical zone of "missing magnetisation" which smooths the measured field in the borehole, but the measured axial field anomaly in the borehole asymptotically approaches the theoretical step change as the sensor penetrates a distance equivalent to several borehole diameters into the sheet. The corresponding gradient anomaly has a large amplitude sharp peak around the interface, which drops to negligible values over a distance of several borehole diameters, as a uniformly magnetised sheet has no internal gradient.