Acceleration of 3D potential field data inversion using a BB iterative algorithm

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SUMMARY

As the big-data age arrives, the efficiency of three dimensional inversions of potential field data can be paid enough attention by scholars. A new inversion method is considered to deal with the large potential field data with a fast rate of convergence. Hence, in this paper, a fast inversion method is researched. And the gravity data is used as an example of potential field data to test the efficiency of our inversion method. To achieve this aim, the study region will be divided into huge amounts of rectangular prisms with unknown constant physical properties of rock. The traditional smooth inversion method is the main principle, and a new Barzilai-Borwein iterative algorithm is applied to ensure the rapid rate of convergence of the inversion method. To compare the rate of convergence of the BB (Barzilai-Borwein) iterative algorithm, the iterative gradient descent algorithm and the iterative conjugate gradient algorithm are used as the compared algorithms. To test high efficiency of the new fast developed inversion method, the large synthetic gravity data are performed. The contrast analysis results can easily reflect the high efficiency of our new inversion method. The great practical value of our inversion method is expounded by a real gravity data. Therefore, the new inversion method may have a great influence on the potential field data inversion.

Key words: the inversion of potential field data, fast convergence, Barzilai-Borwein iterative algorithm

INTRODUCTION

Potential field data is very important to the resources exploration, geological structure research and military filed applications. Due to the emergence of reliable airborne geophysical instruments, the thousands of square kilometers with thousands of line kilometers can be easily covered and the acquisition of huge potential field data may consume little time. These can effectively enhance the application value of the potential field data (Dransfield and Zeng, 2009; Dransfield and Christensen, 2013; Hammer, 1982; Hammond and Murphy, 2003; Lee, 2001). 3D inversion is an important interpretation method, and its main purpose is to obtain the 3D physical property distributions of the sub-surface rock. The solution to massive data processing has already limited the inversion efficiency of the potential field data, which are acquired by the airborne data collection platforms.

In the generalized inversion methods, the sub-surface of the earth model should been discretized into an array of cells with constant rock physical property. As the observed potential field data is inevitably contaminated by noise and the number of observation points is limited comparing with the sub-surface parameters, we will introduce the regularization methods to obtain the best plausible solutions from the infinite number of geophysical equivalent solutions. And the solution conforms to the geophysical and geological requirements. Over last years, many inversion methods have been studied to solve these problems (Barbosa and Silva, 1994; Bosch, 2001; Commer, 2011; Dias, 2009; Dutra and Marangoni, 2009). These inversion methods can improve the resolution of the inversion results. These inversion methods will be of profound significance to the potential field data processing. However, the efficiency of inversion method can also restrict the development of the potential field.

Over the last decade, most of the computer acceleration technology has been achieved and applied by hardware parallelization according to the more and more processor cores into the Central Processing Unit (CPU) and Graphics Processing Units (CPUs) (Chen et al., 2012; Čuma and Zhdanov, 2014). These inversion methods can obviously enhance the inversion efficiency by the hardware parallelization. They are based on the computer hardware. And the numerical algorithms have also been considered to accelerate the inversion processing. They are respectively forward matrix compression (Meng et al., 2016b) and the efficient preconditioner methods from the calculated forward matrix (Meng et al., 2016a). These methods can improve the efficiency of inversion process obviously.

In this paper, a classically numerical algorithm will be applied to improve the inversion efficiency. We will adopt a gradient-descent optimization technique firstly researched by Barzilai and Borwein (Barzilai and Borwein, 1988). And this technology has been successfully applied in many domains (Wang and Ma, 2007; Wang and Yang, 2010). Improving the computational efficiency from the viewpoint of mathematical optimization is the main idea of this algorithm. In the each iteration, a lower residual is obtained comparing with the previous step. Therefore, it can converge to a low overall value in less overall iteration. And in this paper, it is the first time introduced into the 3D potential field data inversion. The theory and numerical and actual tests will be particularly introduced in this paper.

METHOD AND RESULTS

We will give a brief introduction of our methodology which includes potential field data forward modeling and inversion method. The detailed researches are referred to the interested readers. When calculating the potential field response b (N), the sub-surface space of the research area will always be divided into regular prisms with constant rock physical property x (M). With the volume integrals over the potential's Green function, the forward modeling calculated functional can be regarded as the analytical or numerical functional. In the forward modeling part, the potential forward modeling can be calculated by the matrix-vector product.
Since the relation between potential field data and target rock physical property are linear, corresponding response can be expressed in discrete forms as

$$\mathbf{b} = \mathbf{A}\mathbf{x}, \tag{1}$$

where the elements of A, aij, represent the effect of prism attribute i (i = 1,…, N) at observed data location j (j = 1,…,M). bj represent the observed potential field data at the data j (j = 1,…,M). And mi represent the rock physical property of prism i (i = 1,…, N).
In the inverse process, the measured response data b is used to recovered the unknown rock physical property x. The inversion results are instable and non-unique. Due to these reasons, we will introduce the regularization algorithm to obtain the most geophysical and geologically plausible solutions from the infinite set of mathematical equivalent solutions. The functional Jα(x) with the Tikhonov parameter is minimized (Tikhonov and Arsenin, 1977).

$$\mathbf{J}^{\alpha}(\mathbf{x}) := \varphi(\mathbf{x}) + \alpha\Omega(\mathbf{x}) \rightarrow \min \tag{2}$$

where ":=" means "defined by". φ(m) is the misfit functional between the observed and calculated data, and it will ensure that the calculate data can fit the observed data very well. Ω(m) is the stabilizer functional and α is a regularization parameter that balances the misfit functional and the stabilizer. As the inversion becomes closer to the converged results, the adaptive regularization is used to decrease the stabilizer contribution (Zhdanov, 2002). The important regularization parameter α is choose by the adaptive method which is introduced in detail in this book (Zhdanov, 2002). The corresponding model weights and data weights are respectively introduced based on the integrated sensitivity matrix which ensures equal sensitivity of the sub-surface prims located at different sub-surface positions and data matrix which reduces the noise form the each measured data points. In the following section, the different algorithms to minimize the parameter functional Jα(x) will be discussed. And the inversion basing on the next algorithms proceed until the residual reaches a given threshold or a maximum number of iterations are reached.
The stabilizer will play the core role on the regularization approach. Many geophysicists have paid more and more attentions on the choice of the stabilizer. Such as Smooth stabilizers, they can tend to produce smooth spatial distribution of petro-physical properties of anomalous sources. The seeking minimization of a function of difference between the current model and a priori model is the core idea. And focusing stabilizers are also studied and applied in many searches (Zhdanov, 2002; Zhdanov, 2009). Obtaining the sharper boundaries and petro-physical property contrasts is the main core idea. The principle of focusing stabilizers comes from the idea of minimization of volume of nonzero parts. And in this paper, the acceleration of 3D potential field data inversion is the core. Therefore, the smooth stabilizer is used to verify the performance of our inversion method.

Table 1 new inversion algorithm flow

Step 1. Initialization: Given initial point $\mathbf{x}_0$, tolerance $\rho > 0$, set k = 1; Compute $\mathbf{g}_k$.
Step 2. Check whether the stopping condition holds: If equation $\|\mathbf{g}_{k+1}\| \le \rho\|\mathbf{g}_1\|$ is satisfied, STOP; otherwise, set $\mathbf{s}_k = -\mathbf{g}_k$.
Step 3. Compute a Barzilai-Borwein step: ν_k by equation ν_k^{BB1} or ν_k^{BB2}.
Step 4. Update the current iteration point by setting $\mathbf{x}_{k+1} = \mathbf{x}_k + \nu_k\mathbf{s}_k$ with projection
Step 7. Step 5. If $\|\mathbf{b}_{k+1}\| \le \epsilon$, or k exceeds the maximum iterative steps, or the RMS error < ϵ, output $\mathbf{x}_k$, STOP; otherwise, set $k = k + 1$, GOTO Step 3

The gravity field data can be chose as a typical representation of the potential field data. In this section, a gravity field inversion simulation will be applied to study the characteristics of BB iterative algorithm. In this simulation, the simulated gravity data (input data) is generated by the forward model of rock density. And then, according to the inversion method, the sub-surface rock density can be obtained by processing the input gravity data. In practice, the observed data b cannot be obtained exactly. The observed data is usually contaminated by different kinds of noise, e.g. Gauss form. The noise can be recorded besides the true gravity data. Therefore, to test availability, different levels noise will be added into the true gravity data, e.g. 3%, 7% and 10% noise levels.
To illustrate the computational efficiency, the proposed algorithm has been applied to test on a fine gridded synthetic problem with dimensions 30 × 30 × 12, which yields 10,800 unknown rock densities. The data domain is 3000 × 3000 m with the line spacing and the measuring point spacing is both 100 m. In this text, the distances are given in meters but they can be arbitrary. This synthetic example can test the robustness and acceleration of our proposed inversion algorithm. Figure 1A shows the simulated gravity field data produced by a synthetic model consisting of three horizontal elongated prisms. And Figure 1B give visual view of the corresponding sub-surface spatial distribution characteristics of anomalous sources. The detailed parameters of these models can be found in the Table 2. The inversion results obtained from the potential field data different noise level can reveal the performance of these three algorithms.

Table 2 the simulative sub-surface parameters of the synthetic models

Model position	X×Y×Z dimensions(m)	Center Depth (m)	Residual density (g /cm^3)	Background density (g/cm^3)
Low model	900×500×300	350	1	0
Middle model	900×400×300	450	1	0
Deep model	1000×400×300	550	1	0

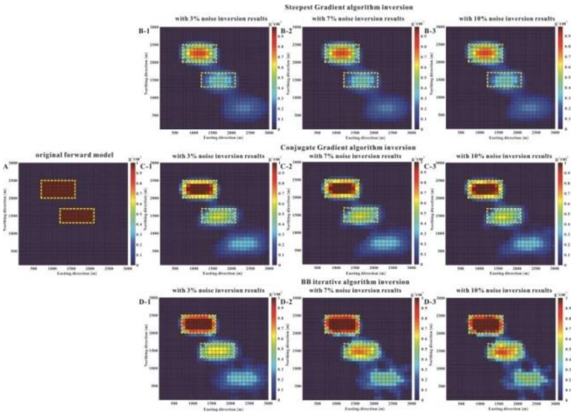

Figure 1: The horizontal cross sections of a density distribution derived from the inversion with a steepest descent algorithm, CG algorithm and BB iterative algorithm applied to the data in Figure 2. Horizontal sections correspond to layer at the depth of 300 m. The simulated models are shown in (A). The inversion results with three algorithms can be found in (B), (C), (D), respectively. And the each figure -1, -2 and -3 respectively represent the inversion results from the gravity data with 3%, 7% and 10% noise levels.

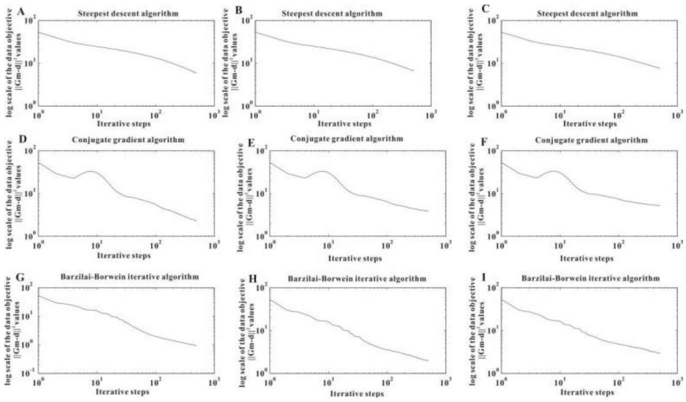

Figure 2: The decreasing of the data objection function with three different algorithms adopt the difference noise levels. A-C applies the steepest descent algorithm, D-F apply the CG algorithm and G-I apply the BB iterative algorithm. The inversion results with 3% noise level are A, D and G, 7% noise level are B, E and H and 10% noise level are C, F and I.

The inversion results of numerical experiment can be found on figure 1. The compared results of three algorithms can show the inversion results of these three inversion algorithm. And the figure 2 can show the advantage of this new inversion algorithm. It can improve the efficiency of 3D inversion. This new inversion method proposed in this paper can effectively solve the disadvantage that the large scale of data may consume lot of times.

CONCLUSIONS

The benefits of the 3D potential field data inversion can obtain the sub-surface physical property distribution of the anomalous rock. From these inversion results, the sub-surface geological and geophysical information can be obtained. However, with the arrival of the era of big airborne data, 3D potential field data inversion faces the large challenge. How to inverse the large airborne data quickly? It is an important for geophysicists. Considering this problem, a fast inversion method using BB iterative algorithm has been applied in potential field data inversion. The conventional algorithm will consume the large inverse time, such as steepest descent algorithm and CG algorithm. From the numerical and read data test, the new inversion method will be an effective method to accelerate the inversion process. It can save lots of inversion consumed time. Therefore, this new algorithm that can reduce the number of iteration and computer time is required to solve the 3D potential field data inversion problem.

The BB iterative algorithm detailed here, which is introduced to solve the ill-posed problem and now to solve the inversion problem, has its useful and significant properties. It can ensure that the inversion process has a quick convergence rate and allow straight-forward implementation of regularization schemes. The synthetic example shows the power of this particular gradient-descent method and large-scale implementation of this technique is straight-forward. The real data inversion test can demonstrate the practical application effect. These tests show the significant application foreground, especial for the large airborne data era.

The advantages of the BB iterative algorithm are obvious with its simplicity of implementation, fast convergence rate and extremely low memory requirements. The study motivation of BB iterative algorithm is to solve the well-posed quadratic programming problems. This problem has been researched in many years. In the next application of this algorithm, the researchers find that it can also applicable to ill posed problem. It has been demonstrated to be an effectively regularization method. And then we apply this algorithm into 3D potential field data inversion. The model and real experiments show that it is an effective tool to solve the inversion problems. With the development of airborne geophysical instruments, the era of big data is here. In the large potential data inversion, the BB iterative algorithm can play a tremendous role in the large-scale and ill-posed 3D potential field data inversion problems.

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