AN OVERVIEW OF TENSORS, GRADIENT AND INVARIANT PRODUCTS IN IMAGING AND QUALITATIVE INTERPRETATION

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SUMMARY

Potential Field Gradient Tensors are a multichannel dataset combining 5 independent components in a matrix array. As such, the data can be used and combined in many ways. A very common problem right across the world of geoscience is that even standard potential field transforms are not actually well understood by users. How does one expand grid transform concepts into the realm of tensors, where so many new combinations and concepts such as Invariants and Phase exist, and create lasting basis for industry interpretation?

It is important that all images used in potential field analysis carry some sort of physical meaning which is understood by the interpreter. True understanding arises from geophysically modelling a known 3D geological model, creating the grid transforms from the forward response of the model, and comparing these to the geology.

3D forward gravitational responses of a 3D model of a simple two-body basin-basement system with conjugate faulting and a dome-basin shape are used to generate the examples. Depths to the Basin-Basement interface were computed from the model and are presented as grids and contours draped on the gravity gradient imaging products to illustrate their responsiveness to the basement architecture.

Various combinations of traditional gravity and its gradient transforms, as well as tensor invariants and phase products, are assessed against the model. It is shown that certain imaging products show more responsiveness to physical property variations, whilst others are more sensitive to geometry, but combining these in novel ways can approach understanding of subsurface mapping possibly not explored previously using potential fields.

Key words: Tensors, Gradients, Interpretation, Imaging, Transforms.

INTRODUCTION

Gravity Gradient Tensors in exploration geophysics are a multichannel dataset combining 5 independent components in a matrix array (Figure 1). Six of these components, the diagonal components Gxx, Gyy, Gzz, and the rotational components Gxy, Gxz, Gyz, are routinely delivered by acquisition contractors as interpretable data channels, which may be used and combined in many ways. It is common when finding descriptions of potential field interpretation datasets in reports to read terms such as "Vertical Gravity", "First Vertical Derivative", "Horizontal Gradient", or "Analytical Signal". These are combinations or derivations of the components of the Gravity Gradient (or Magnetic) Tensor. A very large problem throughout the exploration industry (particularly in oil) is that sometimes these are not actually even understood by the users, just used to see an image enhancement or an edge of some feature.

It is important that all images used in potential field analysis carry some sort of physical meaning which is understood by the interpreter. Despite this, many elaborate filters and transforms are commonly used and delivered which don't convey any physical meaning at all. To truly understand what an anomaly means for a particular dataset, you need to model an object in 3D and compute the particular response with differing parameters to alter the anomaly behaviour. However the singular purpose of this paper is to re-emphasise the physical meaning of some of the most common and slightly exotic transforms, and show, with simple image combinations, how meaning can be derived through qualitative interpretation.

Whilst not treading new ground on long-published and standard geophysical imaging techniques, the different properties of tensor gradient products means that illuminating the signals in specific ways, such as certain combinations of intensity bands or colour blending, can help highlight the physical meaning the transform is intended to convey. In particular, the red-green-blue blending of different seismic frequency spectra (eg REF) is growing in popularity and is analogous in many respects to the proposed treatment of gravity gradient products, whereas colour blending of satellite imaging has been used for decades. The most important message is stop viewing gravity gradients as a single band where the only interpretable product is the Gzz, and that it is multiband rich in geological and structural character.

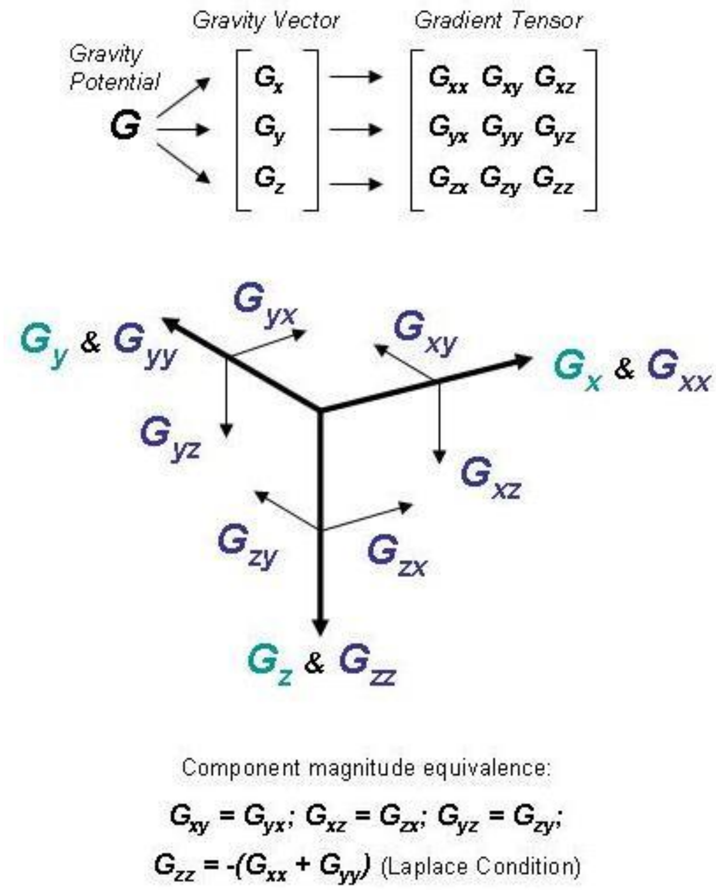

Figure 1: Relationship between Potential Field Vector and Tensor Components.

METHOD AND RESULTS

There is no intention in this paper to re-cover derivations of specific transforms, for that, the reader is encouraged to follow the references. However, I will describe again the gradients, as the description becomes pertinent when I subsequently describe the transforms. What I will then show is the results of synthetic 3D modelling of a Basin-Basement 2-layer model, where all 6 common gradients are computed through forward modelling, and then the interpretative transforms are computed from these. Using this reference model, comparisons can be made to image combinations of the products, so that the physical meaning may be better understood and applied to real datasets. It is not possible to show all full descriptions and images in the extended abstract, so selected examples are described.

Gradient Descriptions

Again I will refer to the tensor components by the diagonal (or axial) components or the rotational components. There is an issue with tensors with regard to reference measurement dependence, which generally pertains most strongly to horizontal gradients. Whilst this can be partially mitigated by the choice of direction in which the survey is flown, the following descriptions regarding the signal strength and behaviour of the components usually apply:

- **Gzz** - is always the strongest signal, but also carries the most geological and acquisition noise. Is usually relatively *Invariant* in its behaviour, which means being the vertical signal it has the lowest observation direction dependence and is most reliably interpreted as a "stand-alone" tensor component.

- **Gyz, Gxz** - these are the horizontal gradients of the gravity with respect to the vertical. These components are equivalent in nature to X and Y derivatives of a standard vertical gravity or magnetic field measurement. They measure the edges of bodies at depth and are often combined together to form a measure called the Total Horizontal Gradient, which is a Pythagorean measure of the strongest horizontal gradient signal: THG = $\sqrt{(Gxz^2 + Gyz^2)}$. This is sometimes referred to as a 2D Analytical Signal. Generally they can be the strongest signal measure after Gzz, but this may depend on the orientation of the geology to the plane, as to which carried the strongest signal. The THG, however, is Invariant about the z-axis, and so because this value remains the same no matter how it is measured, is a preferred interpretative tool for edge mapping and Worming (multi-scale edge detection).

- **Gxx, Gyy** - these are the horizontal gravity gradients in the cardinal directions. They may vary in signal strength depending on the way they are measured, and also can change substantially their anomaly patterns depending on this also. The sum of these effects is equal in magnitude to the Gzz (Laplace Equation) and thus Invariant also, but the strength of each component does not have to be equal. Normally these wouldn't be used for qualitative interpretation until converted to an Invariant form (next section).

- **Gxy** - this is always the weakest signal as it has the lowest coupling with subsurface bodies, but it maps the corners of a vertical body with an opposing anomaly pattern and usually carries a NW-SE/NE-SW fabric on its anomaly pattern. It is strongly geometrically sensitive and rotation-dependent.

It is mathematically possible to transform any one component into any of the other 5 independent components using Fourier Transformations, assuming the potential field properties of a tensor hold (Laplace Equation). However, doing so comes at a cost of introduced noise and loss of signal magnitude compared with a direct measurement.

3D Modelling

The combinations of tensor gradient products are described directly with reference to 3D forward theoretical gravitational responses of a 3D model of a simple two-body basin-basement system (Figure 2). GeoModeller software was used to create the model, which also contains conjugate fault warping. Densities used in contrast for forward modelling are 2.67 g.cm^{-3} for the Basement and 2.4 g.cm^{-3} for the Basin. Depths to the Basin-Basement interface were computed from the model and are presented as grids and contours draped on the gravity gradient products to illustrate their responsiveness to the basement architecture.

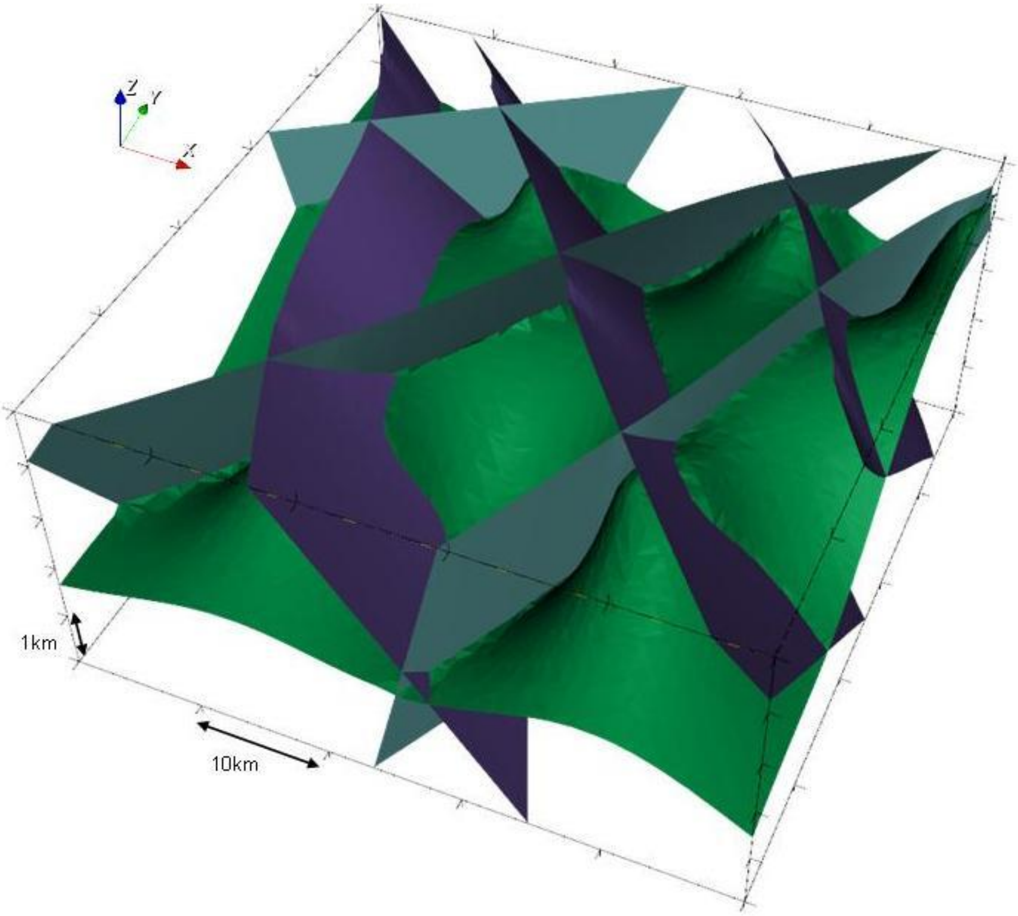

Figure 2. 3D Model illustrating a simple Basin/Basement Interface warped by conjugate faults. Model is 50x50x5 km in dimensions.

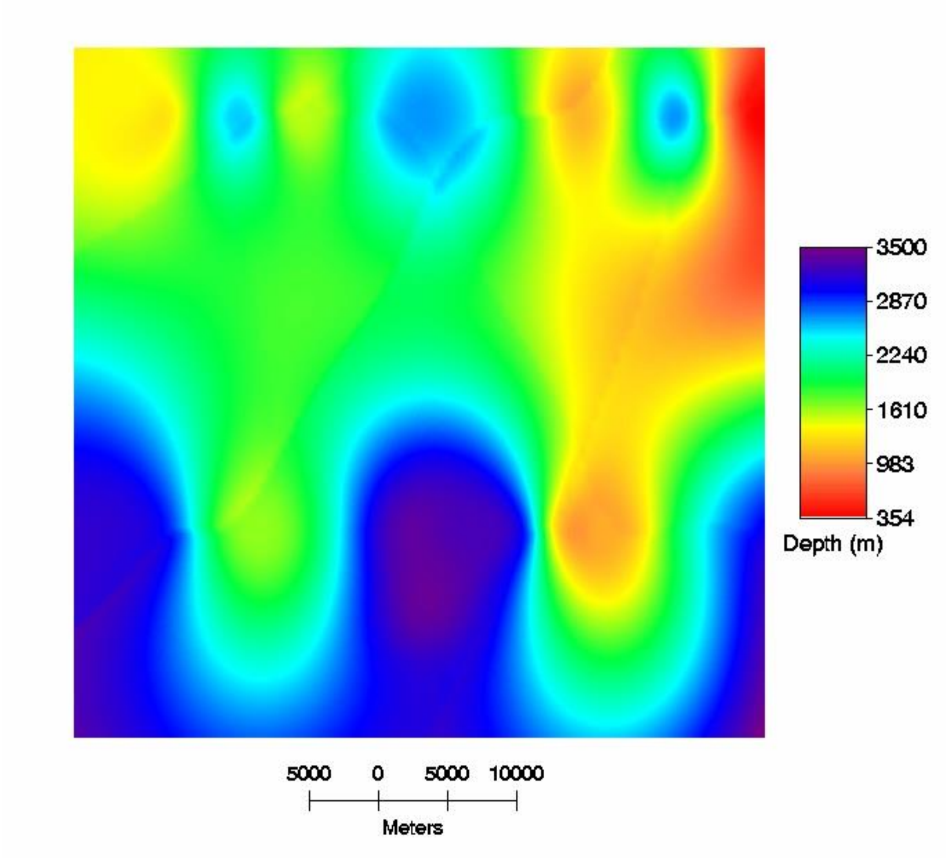

Figure 3. Grid map of Depth to Basement in metres.

Basement Interface grid is a relatively simple example of domes and basins with some offset faulting, mostly in the subvertical planes.

Gradients, Invariants and Combination Images:

The reader is referred to the following sources for details on these transform descriptions: (Intrepid Geophysics, 2017); (Pedersen & Ramussen, 1990); (Verduzco *et al*, 2004).

- **Gzz** - Vertical Gravity Gradient

The Gzz, in absence of near-surface geology effects and significant structural changes, shows a sharper, fairly one-to-one representation of the basement which is easy to interpret. In practice, the Gzz can often be highly contaminated by shallow geology or residual terrain effects, making it more difficult to interpret than expected, and require low-pass filtering (although this has dangers).

- **THG** - Total Horizontal Gradient

Repeating the earlier description, the Total Horizontal Gradient is a Pythagorean measure of the strongest horizontal gradient signal: $THG = \sqrt{(G_{xz}^2 + G_{yz}^2)}$. This is sometimes referred to as a 2D Analytical Signal. The THG is *Invariant* about the z-axis, and so because this value remains the same no matter how it is measured, is a preferred interpretative tool for edge mapping and *Worming* (multi-scale edge detection). The anomaly pattern is strongest where the slopes of the basement are steepest, and weak where there is little net change in the geology, in the saddle zones and at the peaks and troughs of structures. On its own though, it is also clear that the anomaly trends are not completely replicating all of the edge and structural effects. So although this is used as a primary edge detection map, caution needs to be applied in interpretation, and also this type of anomaly map, which is often filtered and used as a product, does not show primary anomalies but edges.

- **THC** - Total Horizontal Curvature

Total Horizontal Curvature is a product measured directly by the purely horizontal components, Gxx, Gyy and Gxy, and thus is measured naturally by the Falcon AGG system. $THC = \sqrt{(G_{uv}^2 + G_{xy}^2)}$ where Guv = (Gyy-Gxx)/2. Loránd Eötvös had only this quantity to work with when experimenting with the torsion balance and successfully interpreted the areas expressing the strongest curvature as salt domes in Germany. The THC can be seen to reflect the areas of strongest geometrical curvature, peaking near the maxima of the basement peaks. However note that it also shows a strong response in some of the most curved basin areas, and is almost zero over the upraised area to the northeast. On its own, it is not sufficient to positively identify all basement highs - it depends on their geometry. A comparison of the THG and the THC, which are both invariant products, shows that they exhibit some complementary features.

- **EVASA** - Analytical Signal Amplitude of Eigenvalues = Square Root of First Invariant (I1)

The Analytical Signal has already been referred to as being in common use, primarily in magnetics interpretation. It is also called the Total Gradient, or Total Gradient Intensity, as an analytical signal is an envelope function containing the amplitude of several signals. The usual expression for the TMI ASA is a sum of squares of the vertical and horizontal derivatives. If we consider an example of a Pole-Reduced TMI, then like the Bouguer or Free Air Gravity, we observe the vertical component of the field. This means that the analogous expression for the gravity gradient tensor would be similar to the Total Horizontal Gradient, with the vertical term added. This would look like this: $ASA = \sqrt{(G_{xz}^2 + G_{yz}^2 + G_{zz}^2)}$, but this is clearly the total gradient of all the vertical components. How to obtain a total gradient of the full tensor?

It comes down to recognising the symmetry of the gradient tensor and its coordinate frame dependence.

A tensor **T** can be decomposed using diagonalisation into a rotational matrix, its inverse and a purely diagonal matrix. This decomposition, $\mathbf{T} = \mathbf{R}^T \mathbf{D} \mathbf{R}$ produces what are termed the Eigenvectors in the **R**otation matrix and Eigenvalues in the **D**iagonal matrix. The Eigenvalues are gravity gradients which contain the equivalent signal as if the tensor were rotated into an invariant reference frame. Therefore, they contain the maximum signal amplitudes for the tensor. A shorthand notation for the Eigenvalue components is based only on their signal variance, being Maximum, Middle and Minimum quantities, so the terms Emx, Emd, and Emn are used. An Analytical Signal Amplitude, or Total Gradient, of these eigenvalues is defined as: $EVASA = -0.5*\sqrt{(Emx^2 + Emd^2 + Emn^2)}$. This quantity is actually the same as the square root of the 1st Tensor Invariant derived from the characteristic polynomial of the tensor.

The envelope behaviour can be seen in the EVASA where all the strong gradients are converted to positive responses under the square and square root signs. The benefit of this behaviour is that it is invariant and shows the strongest magnitude of the overall gradient due to either strongly dense or weakly dense contrasting objects. This can correspond geometrically with strong antiforms or synforms, horsts or intrusions, or even grabens and collapsed structures. Similarly a weak or zero EVASA implies that there is very little geometrical change occurring in this region, which has implications for classifying geology in interpretation.

- **CUBE DET** - Cube Root of Determinant = Cube Root of Second Invariant (I2)

The determinant of a matrix (tensor) is the 2nd invariant in the characteristic polynomial, with the Trace of the tensor being the 0th invariant (or sometimes the first depending on classification), of course the trace of the gravity gradient must be equal to zero by Laplace's Equation. The determinant has the geometrical property of measuring volumes of the imaginary parallelepiped spanned by the vectors defining the tensor, and also of being invariant under scaling (resizing or shrinking). What does this mean for gravity gradients? A determinant from a gravity gradient tensor will have units of $seconds^{-6}$. This doesn't translate easily to thinking of geometry or mass, but it does a scaling of time related to a mass change is worth pondering. The value of the determinant is simply the product of the eigenvalues, ie $det(\mathbf{T}) = Emx*Emd*Emn$. The solution of the characteristic polynomial for the tensor involves the cube root of the determinant, which has the units of Eötvös. This term is invariant and is shown to be more sensitive to the bulk distribution of mass rather than geometry.

Unlike the 1st invariant/EVASA, we have positive and negative changes in gradient which relate to a changing mass accumulation or deficit. The cube root of the 2nd invariant is still sensitive though to structure but tends to show it more under the guise of the apparent density changes. This makes it very useful in combination images.

- **RATIO** - Ratio of I2:I1

Ratio is not so commonly used in interpretation, but possibly plays a better role in combination images. The Tensor Ratio is actually a ratio of I2 to I1, ie the determinant scaled by the squares of the eigenvalues (Pedersen & Rasmussen, 1990). The range is scaled in practice to change from about -1 to + 1, and where it goes to zero the body becomes 2D (ie edges and faults or dykes).

- **TA** - Tilt Angle

The Tilt Angle has increased a lot in recent years as an interpretation tool. Geoscientists primarily use it as a type of Automatic Gain Control tool, helping them see continuations and extensions of anomalies at depth, as well as enhancing structural features and discontinuities in data. This behaviour though is actually a property of how the Tilt Angle is derived, it actually arises from the Analytical Signal concept. Where the Analytical Signal can be thought of as the instantaneous amplitude of a gradient signal, the phase of this signal, which forms what is termed a Hilbert pair with the envelope amplitude, arises from the same calculations to be the angle between the Vertical Gradient and the Total Horizontal Gradient, so it is a phase angle between a vertical and a horizontal vector. This gives the property that the signal ranges from +90° over the top of a vertical mass to -90° at infinity away from the body, with 0° at the edge. This property range causes all signals to have similar relative amplitude regardless if they are weak or strong. Furthermore, the width of the anomaly is depth-dependent. The distance from the zero contour to the ±45° degree contour is also equivalent to the depth to the body, if it is vertical. Generally this is not the case, with asymmetry observed in the shape of TA anomalies - yet this is a dip indicator, usually with the wider side of the anomaly the direction of the dip. Several algorithms have been written to make use of this property for auto-depth estimation. It is worth remembering that a negative vertical gradient produces a negative anomaly, so in the case of magnetics, a reversely (remanent) magnetised unit may produce a negative TA. The ASA will be the discriminator in that situation. Also worth mentioning, for all types of anomalies, depth of burial impacts on the appearance of the anomaly. A deeply buried, tightly folded syncline with dense material could appear as a broad positive anomaly.

- **SPIDX** - Shape Index

The Shape Index (not to be confused with Structural Index SI) is actually a concept which arises from optics studies of curvature of reflective and refractive surfaces. Curvature is a characteristic of geometrical surfaces usually introduced in early studies of analytical geometry, being defined by a normal (radial) vector to the tangent of a curved surface at any point on the surface. For any given location on the surface, one can derive a maximum and minimum curvature from the horizontal gradient data. The Shape Index is a dimensionless quantity which is formed from a ratio of the sums and differences of the maximum and minimum curvature on a gravitational potential surface (C. Cevallos, 2014). The numerator in this ratio turns out to be equivalent to Gzz, and the sum of the denominator is equivalent to the THC. So this is almost an analogy to the Tilt Angle, with THC replacing THG, and with scaling to normalise the radial variation (dimensionless units instead of degrees or radians).

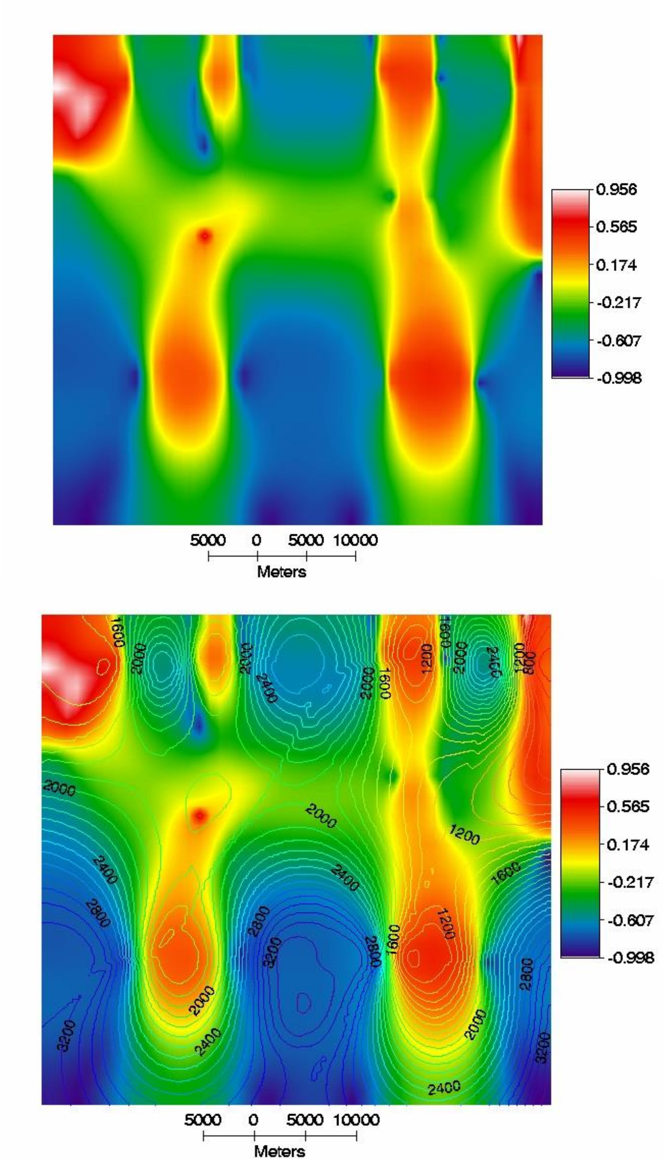

Figure 4. Shape Index (SPIDX) computed from model and overlain depth to basement contours

Indeed in Figure 4 we see sharp adherence of the depth contours to the shape index pattern, possibly even closer match than the TA, although in general the two datasets are clearly showing some similar features. Comparing one dataset to the other in interpretation to examine differences is a useful idea. There is a guide to interpreting the SHAPE of the Shape Index result, as follows: -1 = Bowl, -0.5 = Valley, 0 = Flat, 0.5 = Ridge, 1 = Dome. In magnetics analysis, one might be tempted to perform a Pseudogravity transform after Reduction To The Pole, before computing the Shape Index on the data.

Image Combinations

Images can be interpreted in a large variety of ways. Colour or greyscale palletes, sunshading, colourblending, HSI techniques...the list goes on. Typically the most common type of image display for magnetics and gravity is a rainbow-colour image with histogram equalisation on the colour band and sunshading on the intensity band. There are advantages and disadvantages to many types of imaging, as well as plenty of favouritism, but some general observations are:

- Histogram equalisation looks more attractive, but averages out outstanding anomalies.
- Linear stretch shows anomalies as they are, but may bias detail in an image if there are very large localised high amplitude anomalies present.
- Sunshading enhances structure, but may create directional bias to interpretation depending on the sun angle direction. Dark-shading on non-sunlit slopes of anomalies can mask impression of structure and shape.
- Overly saturated "glossy" images are hard to interpret structurally, but look good in reports and publications.

Combination images can be a good idea, where the image from one channel does not deliver enough information on its own, or is a little too oversaturated in its frequency content and requires something to contrast it. The contrasting dataset, such as an intensity channel, should be the same data as the colour band but imaged in a different way so as to provide the contrast, or be of a related quantity to the main colour band. For example, imaging the TMI with its ASA is a good idea, but mixing gravity and magnetics images is not (unless they are scale independent, but even then tricky).

Where a dataset comes from a multi-channel related source, the opportunity arises to mix the information from the different bands (eg Landsat) and extract useful information from specific colourbands and intensity through RGB or HSI type imaging. Gravity gradients are such a dataset, and certainly any combination of the gradients can reveal certain things, but again, particular combinations work better together.

- **Gz drape Gzz**

This is analogous to a TMI draped on a 1st Vertical Derivative of TMI, Gz takes the colour band and Gzz the Intensity band. When there is a reasonable frequency contrast in the respective grid contents, the Gzz/1vd can provide character and texture analogous to sunangle without the shading, making it useful for contrast interpretation.

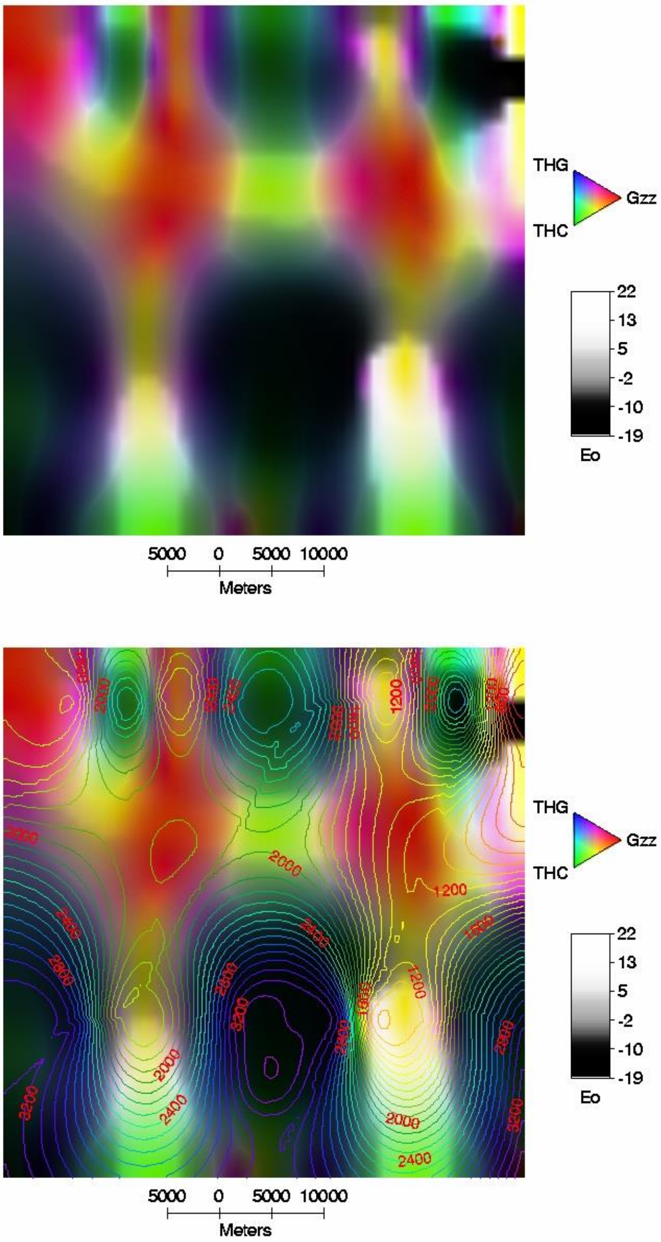

Figure 5. Red-Green-Blue Gzz-THC-THG draped on Cube Det Intensity and depth to basement contours.

- **TA drape EVASA**

The TA set with the EVASA (or ASA for TMI/Bouguer Gravity) as the Intensity band is an extremely useful contrasting dataset, particularly as these two datasets together represent the phase and amplitude of the gradient signals. Although a signal may appear with continuity in a TA, the source may become discontinuous or submerged, or with similar changes in geometrical character or density. The Intensity of the ASA may reveal subtle changes which indicate changes in structure, or show a strong TA continuous signal to indicate flat geometry.

Image intensity is bright where the basin is thick or thin, but weak flat spots do not represent increasing basin thickness, they represent saddle zones in the basement structure. Likewise, the uplifted block in the northwest also appears dark, but because we know it is a high, it tells us that the structure is flat. Additionally, the edges of "shadow" align with the faults. The brightest part of the image is the near-surface high to the northeast.

- **RGB - Gzz-THC-THG**

The selection of these three components to be displayed together as RGB arose from experimentation with tensor data imaging. The complementary nature of these datasets is what drives the RGB colourband choices. Red represents a vertical gradient, whilst the Green and Blue represent the complementary horizontal curvatures. By equalising the distributions, we arrive at a multicolour pattern which accurately represents different geological contrasts and thicknesses (Figure 5).

By examining this image compared to the basement contours, we find that the brightest yellow-white regions correspond to highest density contrast closest to surface on the basement blocks. Blue areas lie on the steep slopes, Dull Green at the bottom of basins, Bright Green on curving slopes, Red on uplifted areas with flatter geology, Orange on transitional highs. There is more to be gained than just this from the image. This only a 2-layer model. Different types of rocks have preferential structural habitats and of course density contrasts. Therefore, this is a type of image which might begin to identify geology.

- **RGB - Gzz-THC-THG drape EVASA (I1**)

A next step is to see if there is more to gain by looking at the intensity of the gradient strength, to assist in characterising structure or geology. This is first trialled using the First Invariant (in fact the square root, the EVASA) as the Intensity band.

This lends a different character to the RGB image which provides some useful information. The dark areas, as in the TA drape EVASA image, show the gradient signal to be weak, with the same conclusions about geometry drawn. Some transitional highs are also lowered in intensity of colour, meaning they could be at comparativly different depths. Central parts of basins are more directly identified. Transitional colour zones may correspond to localised rocks of different character, and structural trends are highly evident.

- **RGB - Gzz-THC-THG drape CUBEDET (I2)**

A second alternative is to use the Second Invariant (cube root) as the intensity band, remembering the physical properties of the determinant. This combination of bands actually lends a more specific geological characterisation to the image, as now the intensity depends more on the changes in density. Therefore the basins look like basins, and the peak intensity zones tend to align specifically with the highs. The basement which is moderately uplifted in the saddles and flat horsts, is well defined. Moderate step-down zones are specifically characterised in green. An edge artefact disrupts the north-east basement high. Transitional colours suggest either thickening or thinning of the sediments, or a different rock type in other scenarios.

CONCLUSIONS

At this point in time, these types of images would represent typical images used by the author as potential field interpretation images. The analogs with magnetics are straightforward, with the caveat of magnetic remanence and preserved field reversal, though true invariant behaviour needs cautious checking and usual rules about dipoles and monopole interpretations apply. However the use of invariance as a discriminator in imaging products should not be understated.

There are probably many other combinations and imaging enhancements available from these datasets and alternative grid transforms and filters. The products chosen are done so because they have a physical meaning that can be applied in qualitative interpretation.

The power of good imaging and pattern recognition by humans cannot be understated. Although industry is moving towards more automatic interpretation, the selection of the right image choices in algorithms will result in better classification and decision making. There are analogues to be made with seismic, satellite and EM imaging from these types of processes described. For example, the Shape Index must have seismic comparisons because of the way it is calculated, and therefore similar application (eg Chen, 2017). Another example is the RGB images of multiband tensors. Filtering (correctly) the tensors to show specific wavelengths, then combining the gradients again as RGB would have a direct analogue in spectral decomposition in seismic imaging. Confusion in imaging choices and lack of understanding of physical properties pertinent to the images, will always lead to poor interpretation.

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